

CURRICULUM, PEDAGOGY AND BEYOND

MAV24
CONFERENCE

5 AND 6 DEC 2024



Calculus Without Algebra Using Primary School Arithmetic and Excel

Presented by Enzo Vozzo on 5 & 6 Dec 2024
Mathematics Teacher of Mentone Grammar

Calculus – Algebra = Arithmetic

G28 CALCULUS WITHOUT ALGEBRA USING PRIMARY SCHOOL ARITHMETIC AND EXCEL.

Subtheme: Pedagogy

Enzo Vozzo, Mentone Grammar
(Year 11 to Year 12)

The fundamental theorem of calculus that connects the two branches of differential and integral calculus is one of the greatest milestones and discoveries in the history of mathematics. Although algebra is used to prove this important theorem, students who struggle with algebra miss out on appreciating how derivatives and integrals are intimately related. However, without any algebra, we can show the fact of this theorem using only the four operations of arithmetic (primary school maths!) and Excel. We will analyse functions such as polynomials, trigonometric, exponential, logarithmic and others by dividing them into 100 (or more) rectangles. Then calculating dy/dx (using subtractions and divisions) and areas (using multiplications and additions), the original function can be reconstructed after differentiation and integration, hence showing the inverse nature of these two branches of calculus.

KEY TAKEAWAYS:

1. A non-algebraic demonstration of the fundamental theorem of calculus.
2. Use of primary school arithmetic (addition, subtraction, multiplication and division) to demonstrate the fundamental theorem of calculus.
3. Use of simple operations in Excel to demonstrate the fundamental theorem of calculus.

Remember: Delegates should be familiar with the fundamental theorem of calculus and have a basic working knowledge of Excel. You are invited to bring you laptop with Excel installed so you can implement the demonstration of this theorem.

$$f(x) \xrightarrow{\quad} \frac{\overset{d}{\cancel{d}}}{\underset{dx}{\cancel{dx}}} f(x) \xrightarrow{\quad} \cancel{\int} \frac{\overset{d}{\cancel{d}}}{\underset{dx}{\cancel{dx}}} f(x) \cancel{dx} \xrightarrow{\quad} f(x)$$

These "cancellations" in a sense hint at the inverse nature of differentiation and integration.

$$f(x) \xrightarrow{\quad} \int f(x) dx \xrightarrow{\quad} \frac{\overset{d}{\cancel{d}}}{\underset{dx}{\cancel{dx}}} \cancel{\int} f(x) \cancel{dx} \xrightarrow{\quad} f(x)$$

Functions that can be analysed:

Polynomial: $f(x) = ax^3 + bx^2 + cx + d$

Trigonometric:
$$\left\{ \begin{array}{l} f(x) = A \sin[b(x - c)] + d \\ f(x) = A \cos[b(x - c)] + d \\ f(x) = A \tan[b(x - c)] + d \end{array} \right.$$

Exponential: $f(x) = Ae^{b(x-c)} + d$

Logarithmic: $f(x) = A \ln[b(x - c)] + d$

Reciprocal: $f(x) = A \frac{1}{b(x - c)} + d$

n th root: $f(x) = A\sqrt[n]{b(x - c)} + d$

Gaussian integral: $f(x) = Ae^{-b(x-c)^2} + d$

Derivative of $\tan^{-1}(x)$: $f(x) = \frac{1}{x^2 + 1}$

Circle: $f(x) = \sqrt{r^2 - x^2}$

Ellipse: $f(x) = \sqrt{b^2 \left(1 - \frac{x^2}{a^2}\right)}$

$x \sin(x)$: $f(x) = x \sin(x)$

The process:

1. Select a function with its parameters.

2. Select the limits of integration, x_{min} and x_{max} .

3. Let $n = 1000$. → Column A

4. List the values of x from x_{min} to x_{max} . → Column B

5. Calculate $dx = \frac{x_{max} - x_{min}}{n - 1}$. → Cell C8

6. Generate the $f(x)$ from x_{min} to x_{max} . → Column C

7. Calculate the values of dx . → Column D

8. Calculate the values of dy . → Column E

9. Calculate the values of dy/dx . → Column F

10. Sum the values of dy (height of each rectangle). → Column G

11. Calculate the *area* of each rectangle. → Column H

12. Sum the *area* of all the rectangles. → Column I, J, K

13. Differentiate the accumulating *area*. → Column L

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		a	b	c	d							
3		0	-1	0	1							
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		-1	0.002002	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	-1	0				0	0	0	1.333332	1.333332	
12	2	-0.998	0.004	0.002002	0.004	1.997998	0.004	8.01E-06	8.01E-06	-----	-----	0.004
13	3	-0.996	0.007992	0.002002	0.003992	1.993994	0.007992	1.6E-05	2.4E-05	-----	-----	0.007992
14	4	-0.99399	0.011976	0.002002	0.003984	1.98999	0.011976	2.4E-05	4.8E-05	-----	-----	0.011976
15	5	-0.99199	0.015952	0.002002	0.003976	1.985986	0.015952	3.19E-05	7.99E-05	-----	-----	0.015952
16	6	-0.98999	0.01992	0.002002	0.003968	1.981982	0.01992	3.99E-05	0.00012	-----	-----	0.01992
17	7	-0.98799	0.02388	0.002002	0.00396	1.977978	0.02388	4.78E-05	0.000168	-----	-----	0.02388
18	8	-0.98599	0.027832	0.002002	0.003952	1.973974	0.027832	5.57E-05	0.000223	-----	-----	0.027832
19	9	-0.98398	0.031776	0.002002	0.003944	1.96997	0.031776	6.36E-05	0.000287	-----	-----	0.031776
20	10	-0.98198	0.035711	0.002002	0.003936	1.965966	0.035711	7.15E-05	0.000358	-----	-----	0.035711
21	11	-0.97998	0.039639	0.002002	0.003928	1.961962	0.039639	7.94E-05	0.000438	-----	-----	0.039639
22	12	-0.97798	0.043559	0.002002	0.00392	1.957958	0.043559	8.72E-05	0.000525	-----	-----	0.043559
23	13	-0.97598	0.047471	0.002002	0.003912	1.953954	0.047471	9.5E-05	0.00062	-----	-----	0.047471
24	14	-0.97397	0.051375	0.002002	0.003904	1.94995	0.051375	0.000103	0.000723	-----	-----	0.051375
25	15	-0.97197	0.05527	0.002002	0.003896	1.945946	0.05527	0.000111	0.000834	-----	-----	0.05527
26	16	-0.96997	0.059158	0.002002	0.003888	1.941942	0.059158	0.000118	0.000952	-----	-----	0.059158
27	17	-0.96797	0.063038	0.002002	0.00388	1.937938	0.063038	0.000126	0.001078	-----	-----	0.063038
28	18	-0.96597	0.06691	0.002002	0.003872	1.933934	0.06691	0.000134	0.001212	-----	-----	0.06691
29	19	-0.96396	0.070773	0.002002	0.003864	1.92993	0.070773	0.000142	0.001354	-----	-----	0.070773
30	20	-0.96196	0.074629	0.002002	0.003856	1.925926	0.074629	0.000149	0.001503	-----	-----	0.074629

1007	997	0.993994	0.011976	0.002002	-0.00398	-1.98599	0.011976	2.4E-05	1.333308	-----	-----	0.011976
1008	998	0.995996	0.007992	0.002002	-0.00398	-1.98999	0.007992	1.6E-05	1.333324	-----	-----	0.007992
1009	999	0.997998	0.004	0.002002	-0.00399	-1.99399	0.004	8.01E-06	1.333332	-----	-----	0.004
1010	1000	1	0	0.002002	-0.004	-1.998	0	0	1.333332	-----	-----	0

13. Chart the following column as 2D column:

14. Chart the following columns as Scatter with smooth lines:

x vs $f(x)$

x vs $\frac{dy}{dx}$

x vs $\int dy = f(x)$

x vs $f(x) dx$

x vs $\int f(x) dx$

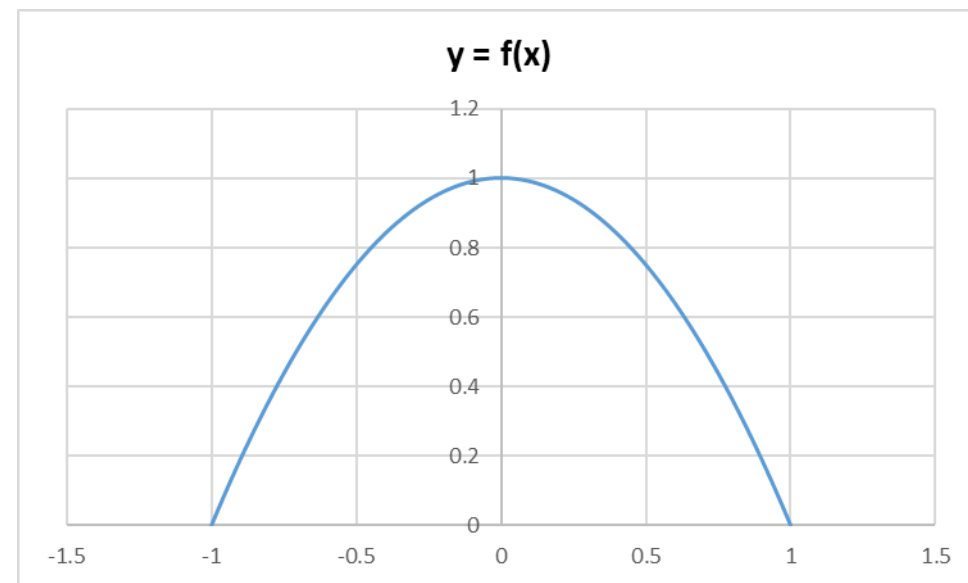
x vs $\frac{d}{dx} \int f(x) dx = f(x)$

15. Change the parameters of the function and the limits of integration and the charts will update automatically.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		a	b	c	d							
3		0	-1	0	1							
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		-1	0.002002	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy/dx	$\int dy = f(x)$	f(x) dx	$\int f(x) dx$	$\int f(x) dx$	$\int f(x) dx$	$\int f(x) dx$
11	1	-1	0				0	0	0	1.333332	1.333332	
12	2	-0.998	0.004	0.002002	0.004	1.997998	0.004	8.01E-06	8.01E-06	-----	-----	0.004
13	3	-0.996	0.007992	0.002002	0.003992	1.993994	0.007992	1.6E-05	2.4E-05	-----	-----	0.007992
14	4	-0.99399	0.011976	0.002002	0.003984	1.98999	0.011976	2.4E-05	4.8E-05	-----	-----	0.011976
15	5	-0.99199	0.015952	0.002002	0.003976	1.985986	0.015952	3.19E-05	7.99E-05	-----	-----	0.015952
16	6	-0.98999	0.01992	0.002002	0.003968	1.981982	0.01992	3.99E-05	0.00012	-----	-----	0.01992
17	7	-0.98799	0.02388	0.002002	0.00396	1.977978	0.02388	4.78E-05	0.000168	-----	-----	0.02388
18	8	-0.98599	0.027832	0.002002	0.003952	1.973974	0.027832	5.57E-05	0.000223	-----	-----	0.027832
19	9	-0.98398	0.031776	0.002002	0.003944	1.96997	0.031776	6.36E-05	0.000287	-----	-----	0.031776
20	10	-0.98198	0.035711	0.002002	0.003936	1.965966	0.035711	7.15E-05	0.000358	-----	-----	0.035711
21	11	-0.97998	0.039639	0.002002	0.003928	1.961962	0.039639	7.94E-05	0.000438	-----	-----	0.039639
22	12	-0.97798	0.043559	0.002002	0.00392	1.957958	0.043559	8.72E-05	0.000525	-----	-----	0.043559
23	13	-0.97598	0.047471	0.002002	0.003912	1.953954	0.047471	9.5E-05	0.00062	-----	-----	0.047471
24	14	-0.97397	0.051375	0.002002	0.003904	1.94995	0.051375	0.000103	0.000723	-----	-----	0.051375
25	15	-0.97197	0.05527	0.002002	0.003896	1.945946	0.05527	0.000111	0.000834	-----	-----	0.05527
26	16	-0.96997	0.059158	0.002002	0.003888	1.941942	0.059158	0.000118	0.000952	-----	-----	0.059158
27	17	-0.96797	0.063038	0.002002	0.00388	1.937938	0.063038	0.000126	0.001078	-----	-----	0.063038
28	18	-0.96597	0.06691	0.002002	0.003872	1.933934	0.06691	0.000134	0.001212	-----	-----	0.06691
29	19	-0.96396	0.070773	0.002002	0.003864	1.92993	0.070773	0.000142	0.001354	-----	-----	0.070773
30	20	-0.96196	0.074629	0.002002	0.003856	1.925926	0.074629	0.000149	0.001503	-----	-----	0.074629

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		a	b	c	d							
3		0	-1	0	1							
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		-1	0.002002	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	-1	0				0	0	0	1.333332	1.333332	
12	2	-0.998	0.004	0.002002	0.004	1.997998	0.004	8.01E-06	8.01E-06	-----	-----	0.004
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15	5	-0.99199	0.015952	0.002002	0.003976	1.985986	0.015952	3.19E-05	7.99E-05	-----	-----	0.015952
16	6	-0.98999	0.01992	0.002002	0.003968	1.981982	0.01992	3.99E-05	0.00012	-----	-----	0.01992
17	7	-0.98799	0.02388	0.002002	0.00396	1.977978	0.02388	4.78E-05	0.000168	-----	-----	0.02388
18	8	-0.98599	0.027832	0.002002	0.003952	1.973974	0.027832	5.57E-05	0.000223	-----	-----	0.027832
19	9	-0.98398	0.031776	0.002002	0.003944	1.96997	0.031776	6.36E-05	0.000287	-----	-----	0.031776
20	10	-0.98198	0.035711	0.002002	0.003936	1.965966	0.035711	7.15E-05	0.000358	-----	-----	0.035711
21	11	-0.97998	0.039639	0.002002	0.003928	1.961962	0.039639	7.94E-05	0.000438	-----	-----	0.039639
22	12	-0.97798	0.043559	0.002002	0.00392	1.957958	0.043559	8.72E-05	0.000525	-----	-----	0.043559
23	13	-0.97598	0.047471	0.002002	0.003912	1.953954	0.047471	9.5E-05	0.00062	-----	-----	0.047471
24	14	-0.97397	0.051375	0.002002	0.003904	1.94995	0.051375	0.000103	0.000723	-----	-----	0.051375
25	15	-0.97197	0.05527	0.002002	0.003896	1.945946	0.05527	0.000111	0.000834	-----	-----	0.05527
26	16	-0.96997	0.059158	0.002002	0.003888	1.941942	0.059158	0.000118	0.000952	-----	-----	0.059158
27	17	-0.96797	0.063038	0.002002	0.00388	1.937938	0.063038	0.000126	0.001078	-----	-----	0.063038
28	18	-0.96597	0.06691	0.002002	0.003872	1.933934	0.06691	0.000134	0.001212	-----	-----	0.06691
29	19	-0.96396	0.070773	0.002002	0.003864	1.92993	0.070773	0.000142	0.001354	-----	-----	0.070773
30	20	-0.96196	0.074629	0.002002	0.003856	1.925926	0.074629	0.000149	0.001503	-----	-----	0.074629

$$\int_{-1}^1 -x^2 + 1 \, dx = \frac{4}{3} = 1.333 \dots$$

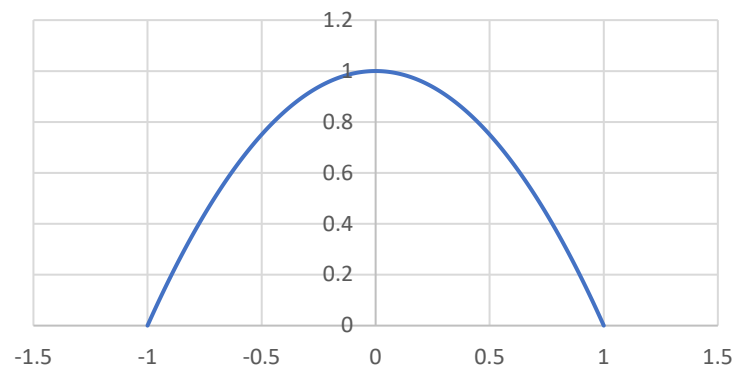


Polynomial functions

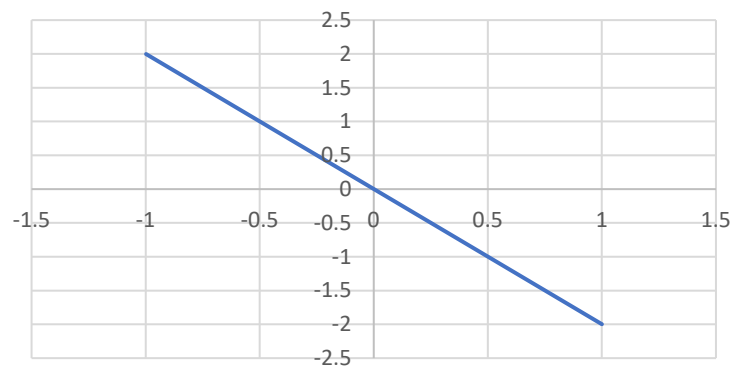
1007	997	0.993994	0.011976	0.002002	-0.00398	-1.98599	0.011976	2.4E-05	1.333308	-----	-----	0.011976
1008	998	0.995996	0.007992	0.002002	-0.00398	-1.98999	0.007992	1.6E-05	1.333324	-----	-----	0.007992
1009	999	0.997998	0.004	0.002002	-0.00399	-1.99399	0.004	8.01E-06	1.333332	-----	-----	0.004
1010	1000	1	0	0.002002	-0.004	-1.998	0	0	1.333332	-----	-----	0

$$f(x) = -x^2 + 1$$

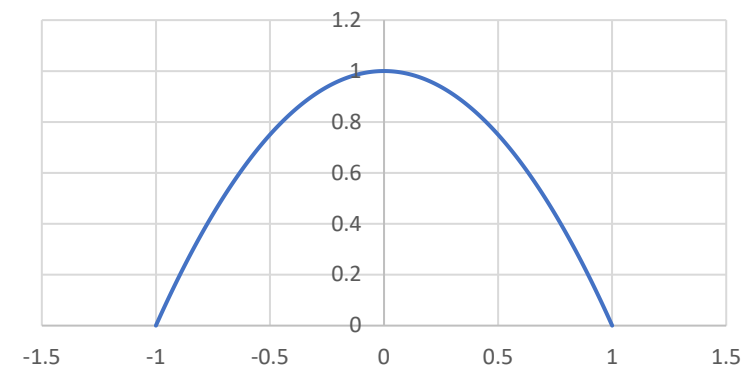
y = f(x)



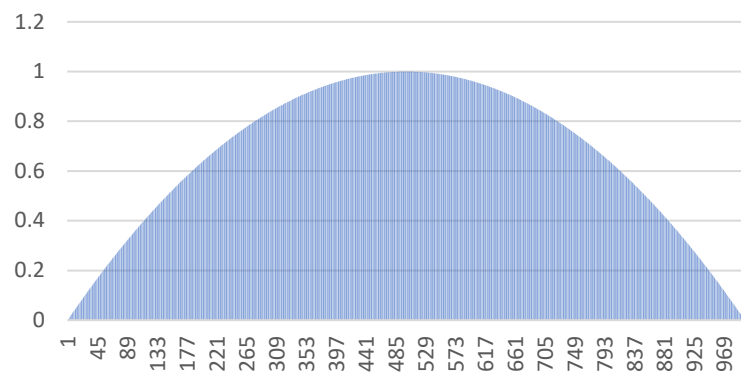
dy/dx



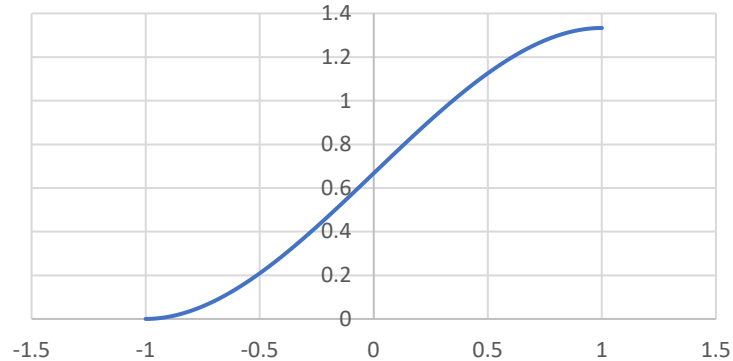
∫ dx = f(x)



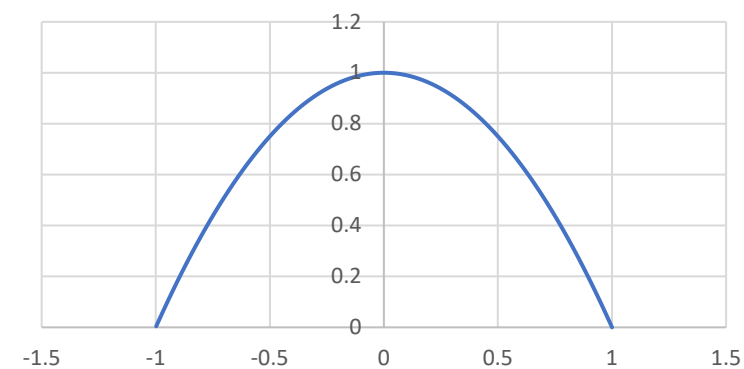
Rectangles of ∫ dy = f(x)



∫ f(x) dx

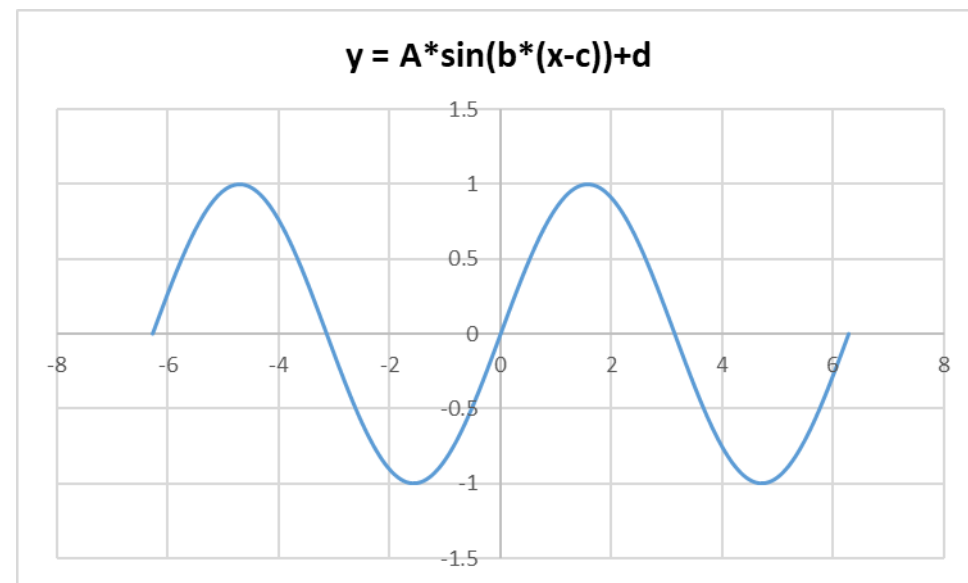


f(x) = d/dx of ∫ f(x) dx



	A	B	C	D	E	F	G	H	I	J	K	L
1							$y = f(x) = A \sin(b(x - c)) + d$					
2		A	b	c	d		$y = f(x) = A \cos(b(x - c)) + d$					
3		1	1	0	0		$y = f(x) = A \tan(b(x - c)) + d$					
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		-6.28319	0.012579	6.283185			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	-6.28319	2.45E-16				2.45E-16	0	0	8.77E-16	8.77E-16	
12	2	-6.27061	0.012579	0.012579	0.012579	0.999974	0.012579	0.000158	0.000158	-----	-----	0.012579
13	3	-6.25803	0.025155	0.012579	0.012577	0.999815	0.025155	0.000316	0.000475	-----	-----	0.025155
14	4	-6.24545	0.037728	0.012579	0.012573	0.999499	0.037728	0.000475	0.000949	-----	-----	0.037728
15	5	-6.23287	0.050295	0.012579	0.012567	0.999024	0.050295	0.000633	0.001582	-----	-----	0.050295
16	6	-6.22029	0.062853	0.012579	0.012559	0.998392	0.062853	0.000791	0.002373	-----	-----	0.062853
17	7	-6.20771	0.075402	0.012579	0.012549	0.997601	0.075402	0.000948	0.003321	-----	-----	0.075402
18	8	-6.19513	0.087939	0.012579	0.012537	0.996653	0.087939	0.001106	0.004427	-----	-----	0.087939
19	9	-6.18255	0.100462	0.012579	0.012523	0.995547	0.100462	0.001264	0.005691	-----	-----	0.100462
20	10	-6.16997	0.112969	0.012579	0.012507	0.994283	0.112969	0.001421	0.007112	-----	-----	0.112969
21	11	-6.1574	0.125458	0.012579	0.012489	0.992862	0.125458	0.001578	0.00869	-----	-----	0.125458
22	12	-6.14482	0.137927	0.012579	0.012469	0.991284	0.137927	0.001735	0.010425	-----	-----	0.137927
23	13	-6.13224	0.150375	0.012579	0.012447	0.989549	0.150375	0.001892	0.012317	-----	-----	0.150375
24	14	-6.11966	0.162799	0.012579	0.012424	0.987657	0.162799	0.002048	0.014364	-----	-----	0.162799
25	15	-6.10708	0.175196	0.012579	0.012398	0.985609	0.175196	0.002204	0.016568	-----	-----	0.175196
26	16	-6.0945	0.187567	0.012579	0.01237	0.983406	0.187567	0.002359	0.018928	-----	-----	0.187567
27	17	-6.08192	0.199907	0.012579	0.012341	0.981046	0.199907	0.002515	0.021442	-----	-----	0.199907
28	18	-6.06934	0.212216	0.012579	0.012309	0.978532	0.212216	0.002669	0.024112	-----	-----	0.212216
29	19	-6.05676	0.224491	0.012579	0.012275	0.975862	0.224491	0.002824	0.026936	-----	-----	0.224491
30	20	-6.04419	0.236731	0.012579	0.01224	0.973038	0.236731	0.002978	0.029913	-----	-----	0.236731

$$\int_{-\pi}^{\pi} \sin(x) dx = 0$$

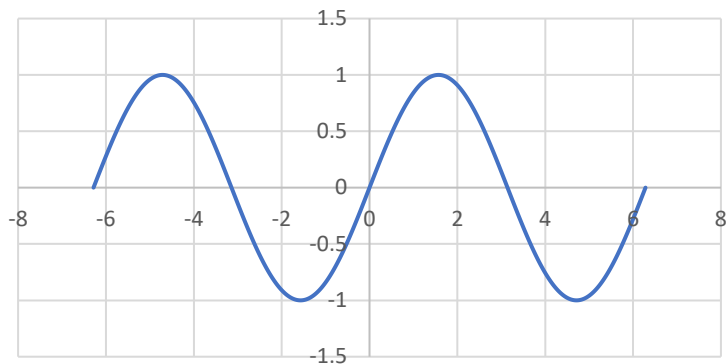


Trigonometric functions:
sin, cos, tan

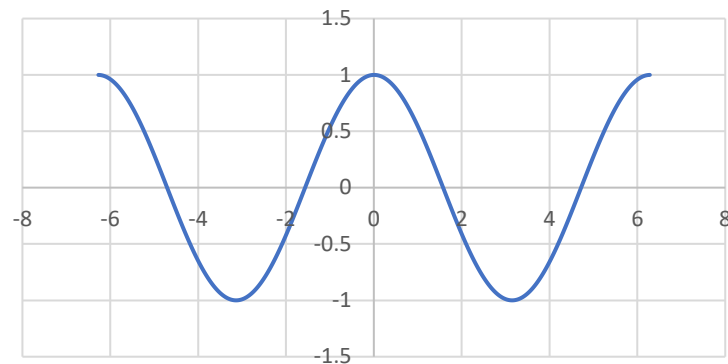
1007	997	6.245448	-0.03773	0.012579	0.012567	0.999024	-0.03773	-0.00047	0.000475	-----	-----	-0.03773
1008	998	6.258027	-0.02516	0.012579	0.012573	0.999499	-0.02516	-0.00032	0.000158	-----	-----	-0.02516
1009	999	6.270606	-0.01258	0.012579	0.012577	0.999815	-0.01258	-0.00016	8.81E-16	-----	-----	-0.01258
1010	1000	6.283185	-2.5E-16	0.012579	0.012579	0.999974	-2.4E-16	-3.1E-18	8.77E-16	-----	-----	-2.5E-16

$$f(x) = \sin(x)$$

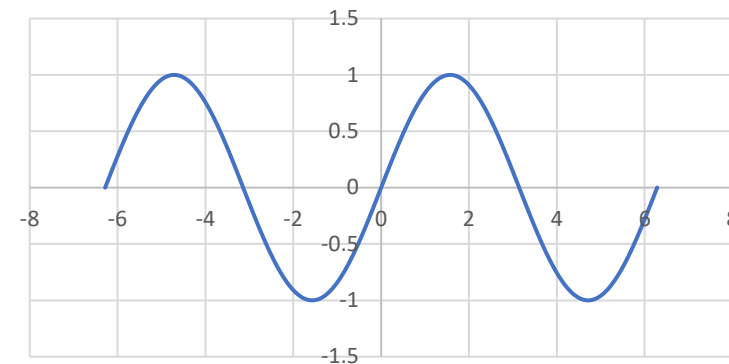
$y = f(x)$



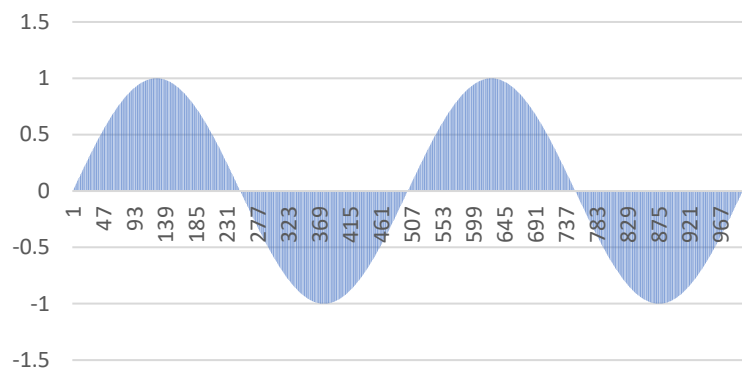
dy/dx



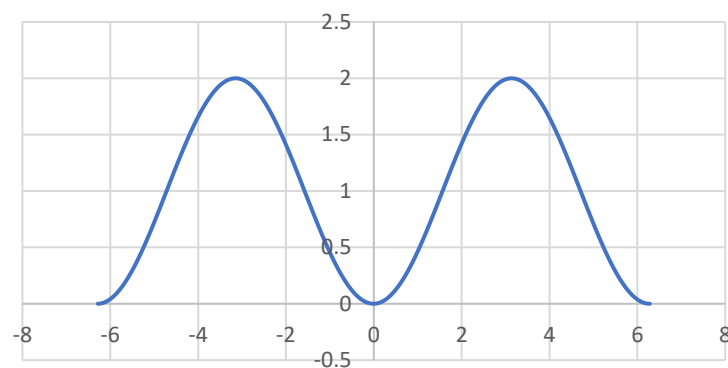
$\int dy = f(x)$



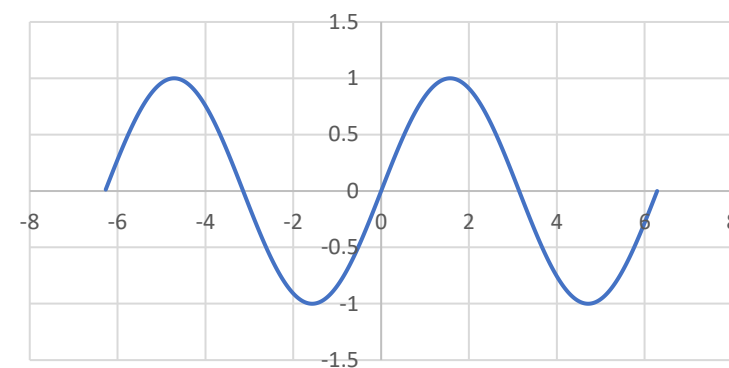
Rectangles of $\int dy = f(x)$



$\int f(x) dx$

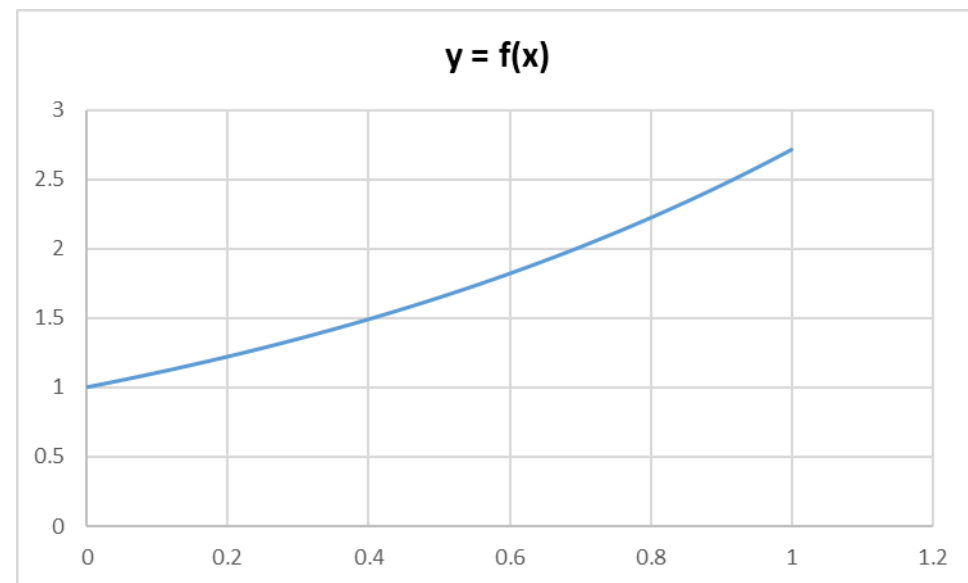


$d/dx \int f(x) dx = f(x)$



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		A	b	c	d		$y = f(x) = A \exp(b(x - c)) + d$					
3		1	1	0	0							
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		0	0.001001	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	0	1				1	0	0	1.719142	1.719142	
12	2	0.001001	1.001002	0.001001	0.001002	1.000501	1.001002	0.001002	0.001002	-----	-----	1.001002
13	3	0.002002	1.002004	0.001001	0.001003	1.001503	1.002004	0.001003	0.002005	-----	-----	1.002004
14	4	0.003003	1.003008	0.001001	0.001004	1.002506	1.003008	0.001004	0.003009	-----	-----	1.003008
15	5	0.004004	1.004012	0.001001	0.001005	1.00351	1.004012	0.001005	0.004014	-----	-----	1.004012
16	6	0.005005	1.005018	0.001001	0.001006	1.004515	1.005018	0.001006	0.00502	-----	-----	1.005018
17	7	0.006006	1.006024	0.001001	0.001007	1.005521	1.006024	0.001007	0.006027	-----	-----	1.006024
18	8	0.007007	1.007032	0.001001	0.001008	1.006528	1.007032	0.001008	0.007035	-----	-----	1.007032
19	9	0.008008	1.00804	0.001001	0.001009	1.007536	1.00804	0.001009	0.008044	-----	-----	1.00804
20	10	0.009009	1.00905	0.001001	0.00101	1.008545	1.00905	0.00101	0.009054	-----	-----	1.00905
21	11	0.01001	1.01006	0.001001	0.001011	1.009555	1.01006	0.001011	0.010065	-----	-----	1.01006
22	12	0.011011	1.011072	0.001001	0.001012	1.010566	1.011072	0.001012	0.011077	-----	-----	1.011072
23	13	0.012012	1.012084	0.001001	0.001013	1.011578	1.012084	0.001013	0.01209	-----	-----	1.012084
24	14	0.013013	1.013098	0.001001	0.001014	1.012591	1.013098	0.001014	0.013105	-----	-----	1.013098
25	15	0.014014	1.014113	0.001001	0.001015	1.013605	1.014113	0.001015	0.01412	-----	-----	1.014113
26	16	0.015015	1.015128	0.001001	0.001016	1.01462	1.015128	0.001016	0.015136	-----	-----	1.015128
27	17	0.016016	1.016145	0.001001	0.001017	1.015637	1.016145	0.001017	0.016153	-----	-----	1.016145
28	18	0.017017	1.017163	0.001001	0.001018	1.016654	1.017163	0.001018	0.017171	-----	-----	1.017163
29	19	0.018018	1.018181	0.001001	0.001019	1.017672	1.018181	0.001019	0.01819	-----	-----	1.018181
30	20	0.019019	1.019201	0.001001	0.00102	1.018691	1.019201	0.00102	0.019211	-----	-----	1.019201

$$\int_0^1 e^x dx = 1.71828 \dots = e - 1$$

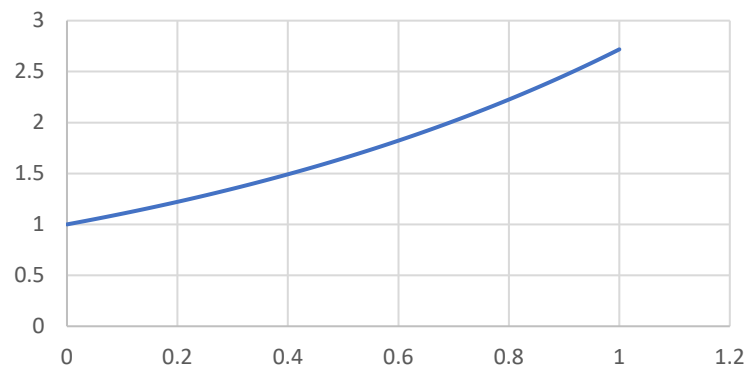


Exponential functions

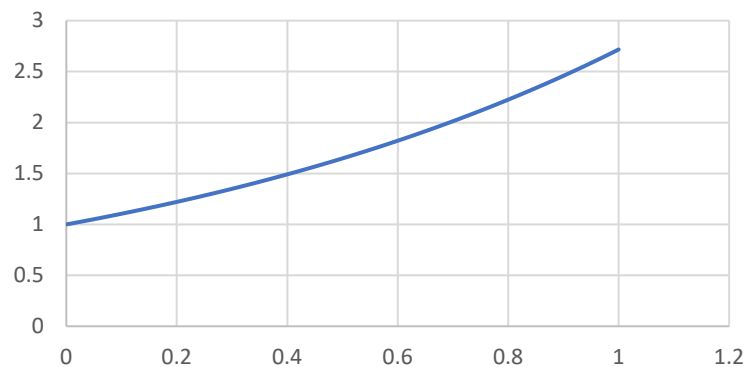
1007	997	0.996997	2.710131	0.001001	0.002711	2.708775	2.710131	0.002713	1.710987	-----	-----	2.710131
1008	998	0.997998	2.712845	0.001001	0.002714	2.711488	2.712845	0.002716	1.713703	-----	-----	2.712845
1009	999	0.998999	2.715562	0.001001	0.002717	2.714204	2.715562	0.002718	1.716421	-----	-----	2.715562
1010	1000	1	2.718282	0.001001	0.00272	2.716922	2.718282	0.002721	1.719142	-----	-----	2.718282

$$f(x) = e^x$$

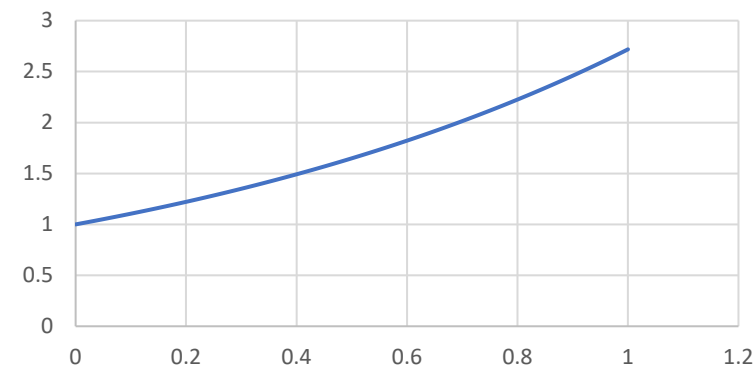
$y = f(x)$



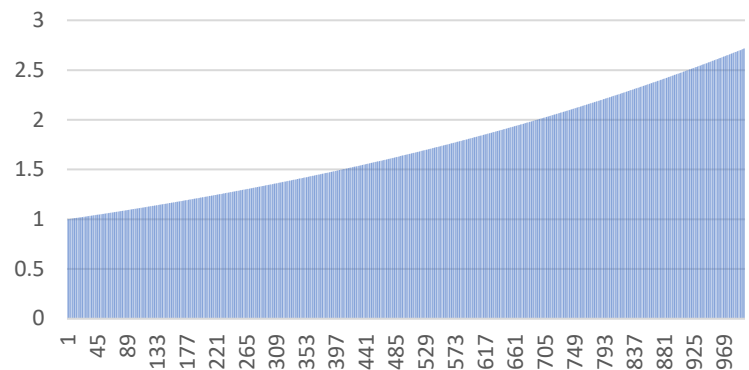
dy/dx



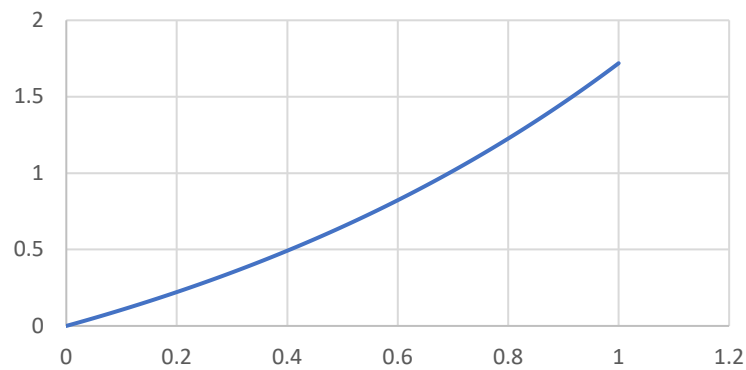
$\int dy = f(x)$



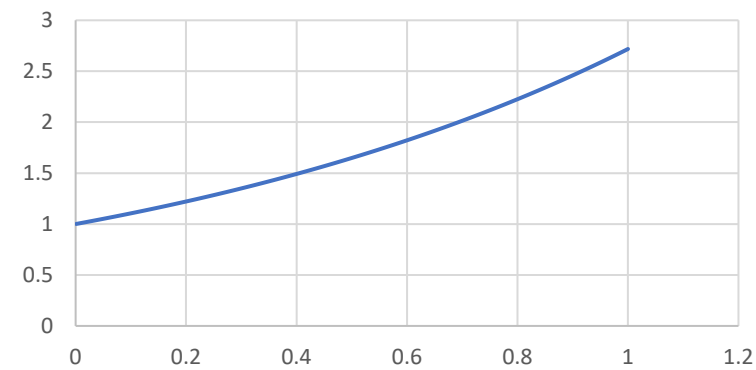
Rectangles of $\int dy = f(x)$



$\int f(x) dx$

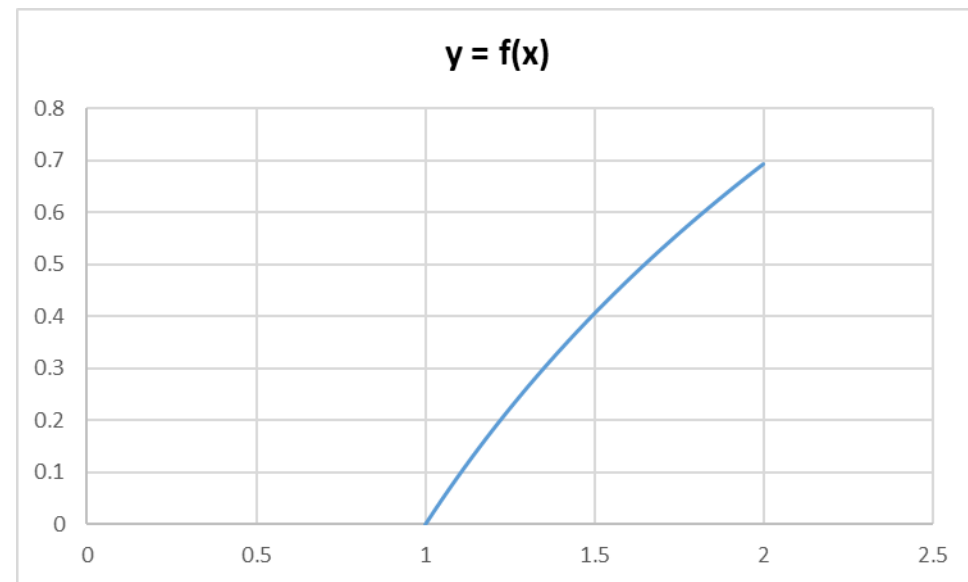


$d/dx \int f(x) = f(x)$



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		A	b	c	d		$y = f(x) = A \ln(b(x - c)) + d$					
3		1	1	0	0							
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		1	0.001001	2			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	1	0				0	0	0	0.386641	0.386641	
12	2	1.001001	0.001001	0.001001	0.001001	0.9995	0.001001	1E-06	1E-06	-----	-----	0.001001
13	3	1.002002	0.002	0.001001	0.001	0.998501	0.002	2E-06	3E-06	-----	-----	0.002
14	4	1.003003	0.002999	0.001001	0.000999	0.997504	0.002999	3E-06	6.01E-06	-----	-----	0.002999
15	5	1.004004	0.003996	0.001001	0.000998	0.996509	0.003996	4E-06	1E-05	-----	-----	0.003996
16	6	1.005005	0.004993	0.001001	0.000997	0.995516	0.004993	5E-06	1.5E-05	-----	-----	0.004993
17	7	1.006006	0.005988	0.001001	0.000996	0.994525	0.005988	5.99E-06	2.1E-05	-----	-----	0.005988
18	8	1.007007	0.006983	0.001001	0.000995	0.993536	0.006983	6.99E-06	2.8E-05	-----	-----	0.006983
19	9	1.008008	0.007976	0.001001	0.000994	0.992549	0.007976	7.98E-06	3.6E-05	-----	-----	0.007976
20	10	1.009009	0.008969	0.001001	0.000993	0.991563	0.008969	8.98E-06	4.49E-05	-----	-----	0.008969
21	11	1.01001	0.00996	0.001001	0.000992	0.99058	0.00996	9.97E-06	5.49E-05	-----	-----	0.00996
22	12	1.011011	0.010951	0.001001	0.000991	0.989599	0.010951	1.1E-05	6.59E-05	-----	-----	0.010951
23	13	1.012012	0.01194	0.001001	0.00099	0.98862	0.01194	1.2E-05	7.78E-05	-----	-----	0.01194
24	14	1.013013	0.012929	0.001001	0.000989	0.987642	0.012929	1.29E-05	9.08E-05	-----	-----	0.012929
25	15	1.014014	0.013917	0.001001	0.000988	0.986667	0.013917	1.39E-05	0.000105	-----	-----	0.013917
26	16	1.015015	0.014903	0.001001	0.000987	0.985693	0.014903	1.49E-05	0.00012	-----	-----	0.014903
27	17	1.016016	0.015889	0.001001	0.000986	0.984722	0.015889	1.59E-05	0.000136	-----	-----	0.015889
28	18	1.017017	0.016874	0.001001	0.000985	0.983752	0.016874	1.69E-05	0.000152	-----	-----	0.016874
29	19	1.018018	0.017858	0.001001	0.000984	0.982784	0.017858	1.79E-05	0.00017	-----	-----	0.017858
30	20	1.019019	0.01884	0.001001	0.000983	0.981818	0.01884	1.89E-05	0.000189	-----	-----	0.01884

$$\int_1^2 \ln(x) dx = 0.38629 \dots$$

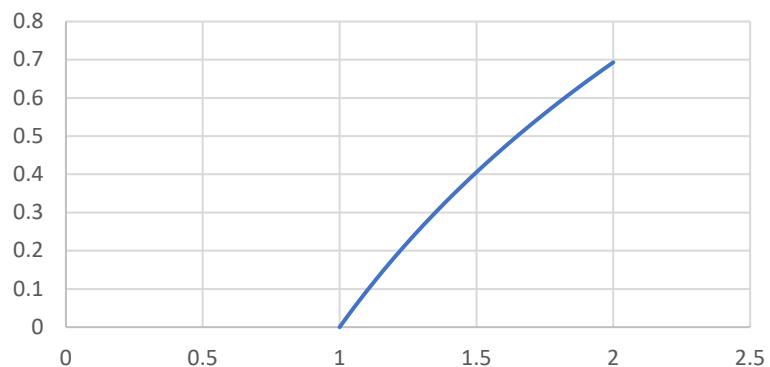


Logarithmic functions

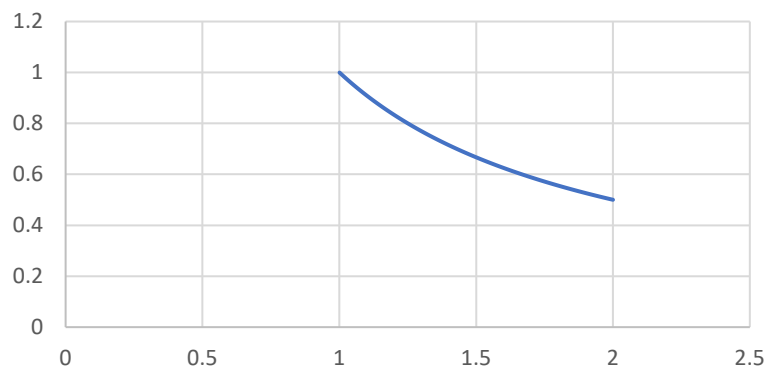
1007	997	1.996997	0.691645	0.001001	0.000501	0.500877	0.691645	0.000692	0.384561	-----	-----	0.691645
1008	998	1.997998	0.692146	0.001001	0.000501	0.500626	0.692146	0.000693	0.385254	-----	-----	0.692146
1009	999	1.998999	0.692647	0.001001	0.000501	0.500376	0.692647	0.000693	0.385947	-----	-----	0.692647
1010	1000	2	0.693147	0.001001	0.000501	0.500125	0.693147	0.000694	0.386641	-----	-----	0.693147

$$f(x) = \ln(x)$$

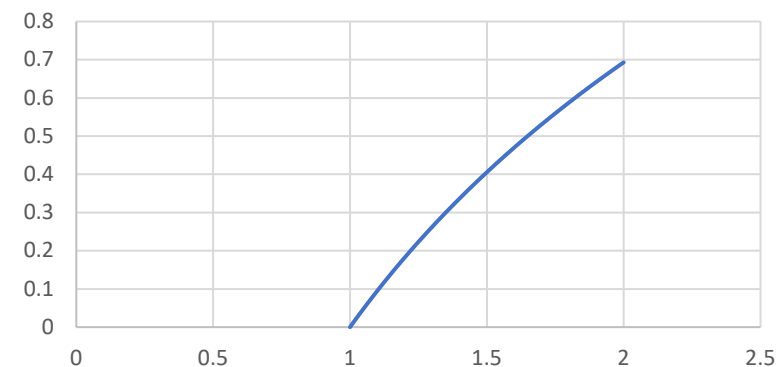
$$y = f(x)$$



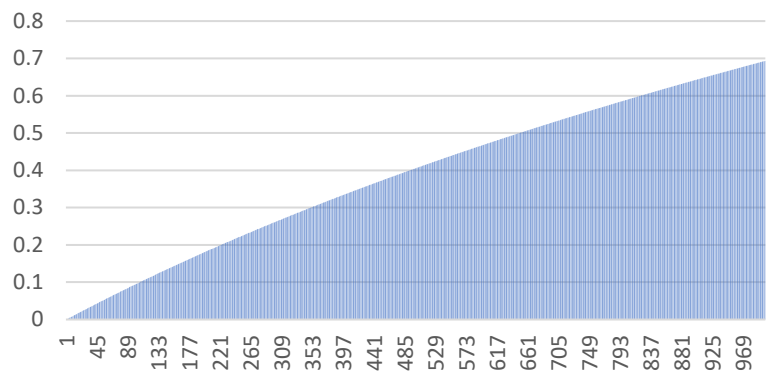
$$dy/dx$$



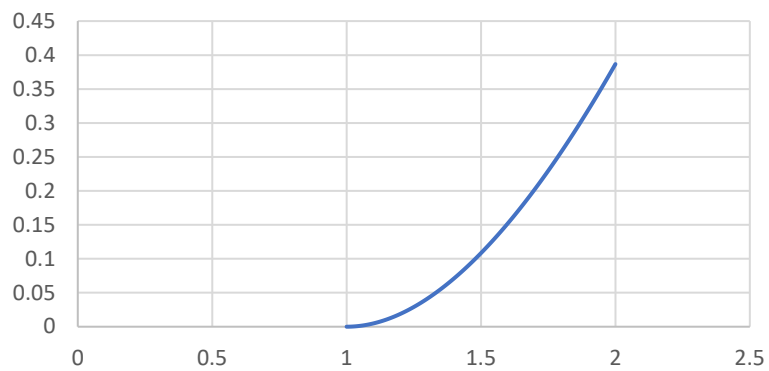
$$\int dy = f(x)$$



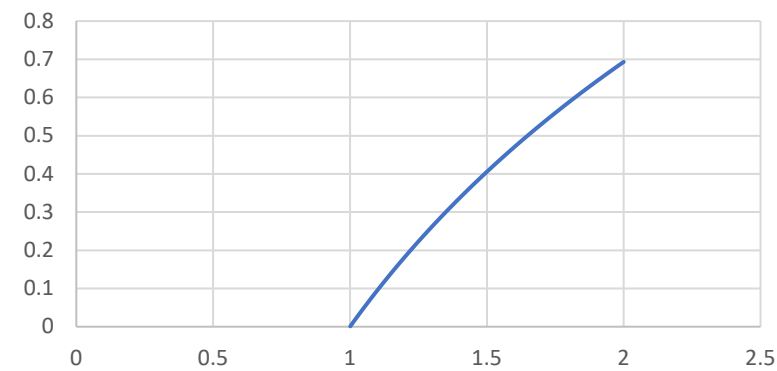
$$\text{Rectangles of } \int dy = f(x)$$



$$\int f(x) dx$$



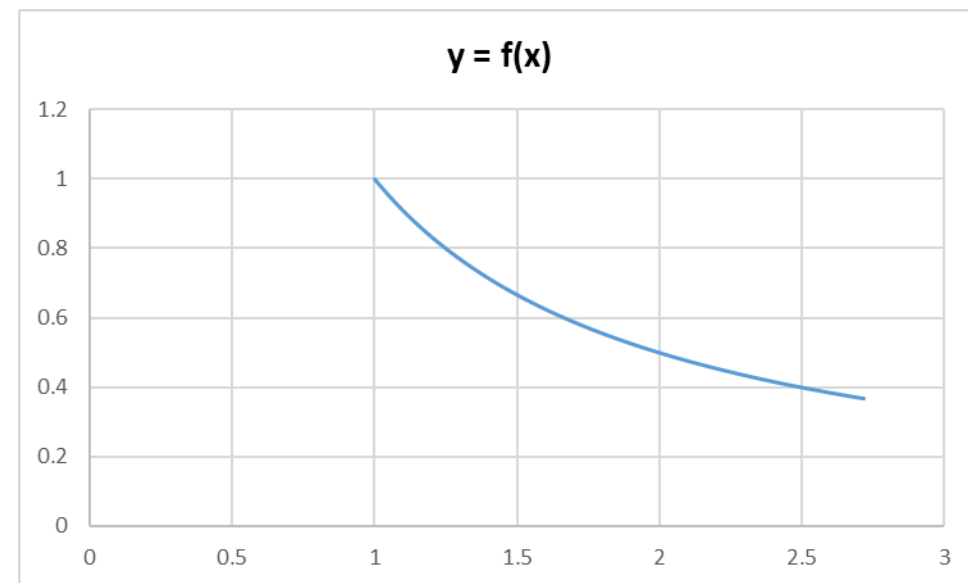
$$d/dx \int f(x) dx = f(x)$$



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		A	b	c	d							
3		1	1	0	0							
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		1	0.00172	2.718282			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	1	1				1	0	0	0.999457	0.999457	
12	2	1.00172	0.998283	0.00172	-0.00172	-0.99828	0.998283	0.001717	0.001717	-----	-----	0.998283
13	3	1.00344	0.996572	0.00172	-0.00171	-0.99486	0.996572	0.001714	0.003431	-----	-----	0.996572
14	4	1.00516	0.994866	0.00172	-0.00171	-0.99146	0.994866	0.001711	0.005142	-----	-----	0.994866
15	5	1.00688	0.993167	0.00172	-0.0017	-0.98807	0.993167	0.001708	0.006851	-----	-----	0.993167
16	6	1.0086	0.991473	0.00172	-0.00169	-0.9847	0.991473	0.001705	0.008556	-----	-----	0.991473
17	7	1.01032	0.989785	0.00172	-0.00169	-0.98135	0.989785	0.001702	0.010258	-----	-----	0.989785
18	8	1.01204	0.988103	0.00172	-0.00168	-0.97801	0.988103	0.0017	0.011958	-----	-----	0.988103
19	9	1.01376	0.986427	0.00172	-0.00168	-0.97469	0.986427	0.001697	0.013655	-----	-----	0.986427
20	10	1.01548	0.984756	0.00172	-0.00167	-0.97139	0.984756	0.001694	0.015348	-----	-----	0.984756
21	11	1.0172	0.983091	0.00172	-0.00167	-0.9681	0.983091	0.001691	0.017039	-----	-----	0.983091
22	12	1.01892	0.981431	0.00172	-0.00166	-0.96484	0.981431	0.001688	0.018727	-----	-----	0.981431
23	13	1.02064	0.979777	0.00172	-0.00165	-0.96158	0.979777	0.001685	0.020413	-----	-----	0.979777
24	14	1.02236	0.978129	0.00172	-0.00165	-0.95835	0.978129	0.001682	0.022095	-----	-----	0.978129
25	15	1.02408	0.976486	0.00172	-0.00164	-0.95513	0.976486	0.00168	0.023774	-----	-----	0.976486
26	16	1.0258	0.974849	0.00172	-0.00164	-0.95193	0.974849	0.001677	0.025451	-----	-----	0.974849
27	17	1.02752	0.973217	0.00172	-0.00163	-0.94874	0.973217	0.001674	0.027125	-----	-----	0.973217
28	18	1.02924	0.971591	0.00172	-0.00163	-0.94557	0.971591	0.001671	0.028796	-----	-----	0.971591
29	19	1.03096	0.96997	0.00172	-0.00162	-0.94241	0.96997	0.001668	0.030465	-----	-----	0.96997
30	20	1.03268	0.968354	0.00172	-0.00162	-0.93927	0.968354	0.001666	0.03213	-----	-----	0.968354

$$y = f(x) = A \frac{1}{b(x-c)} + d$$

$$\int_1^e \frac{1}{x} dx = 1$$

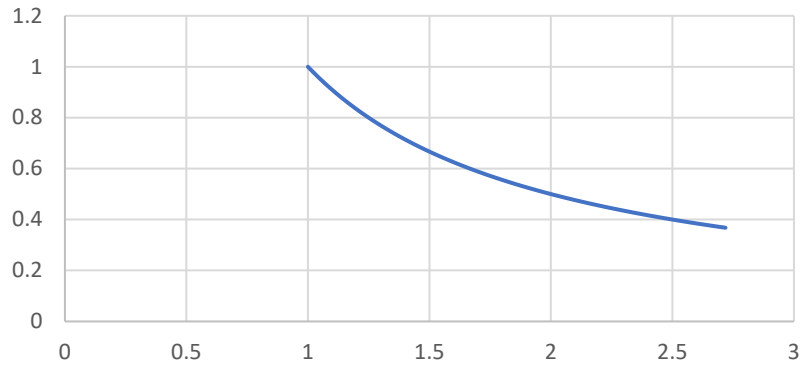


Reciprocal functions

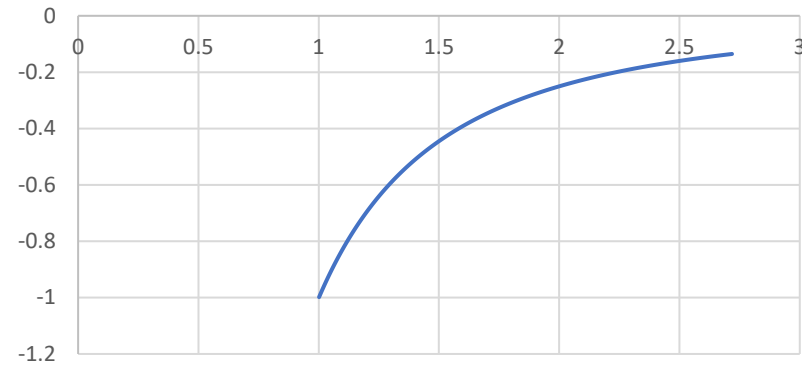
1007	997	2.713122	0.368579	0.00172	-0.00023	-0.13594	0.368579	0.000634	0.997557	-----	-----	0.368579
1008	998	2.714842	0.368346	0.00172	-0.00023	-0.13576	0.368346	0.000634	0.998191	-----	-----	0.368346
1009	999	2.716562	0.368112	0.00172	-0.00023	-0.13559	0.368112	0.000633	0.998824	-----	-----	0.368112
1010	1000	2.718282	0.367879	0.00172	-0.00023	-0.13542	0.367879	0.000633	0.999457	-----	-----	0.367879

$$f(x) = \frac{1}{x}$$

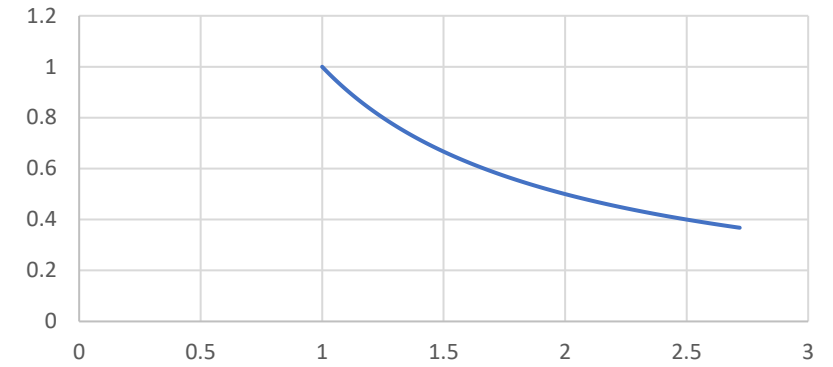
y = f(x)



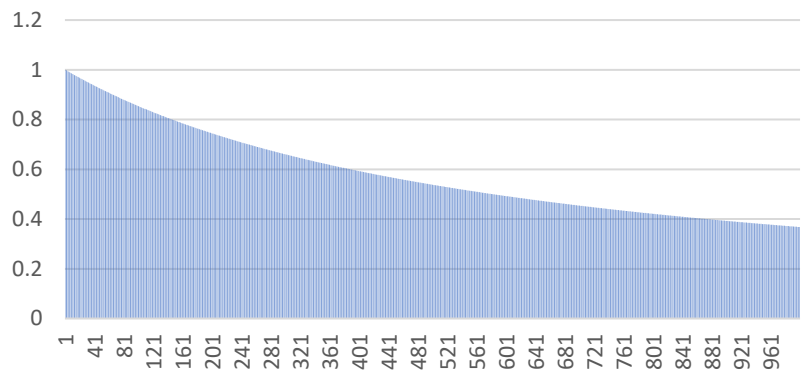
dy/dx



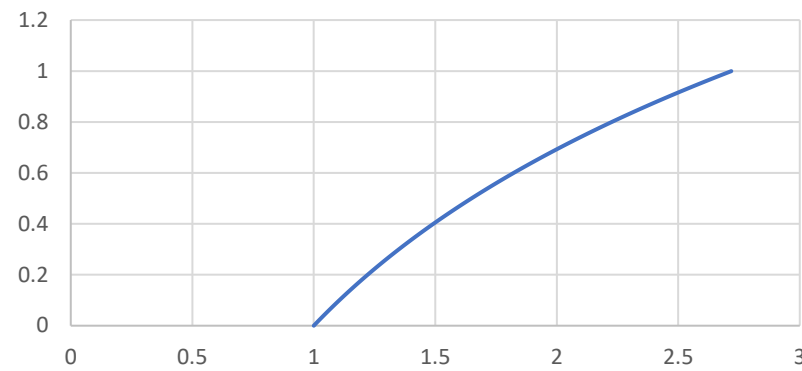
∫ dy = f(x)



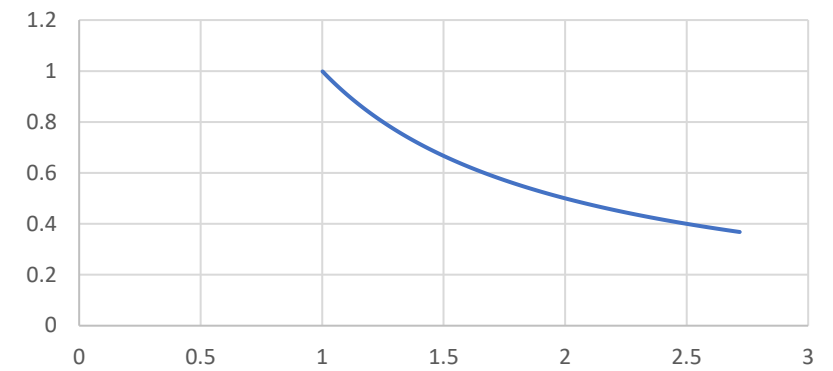
Rectangles of ∫ dy = f(x)



∫ f(x) dx



d/dx ∫ f(x) dx = f(x)

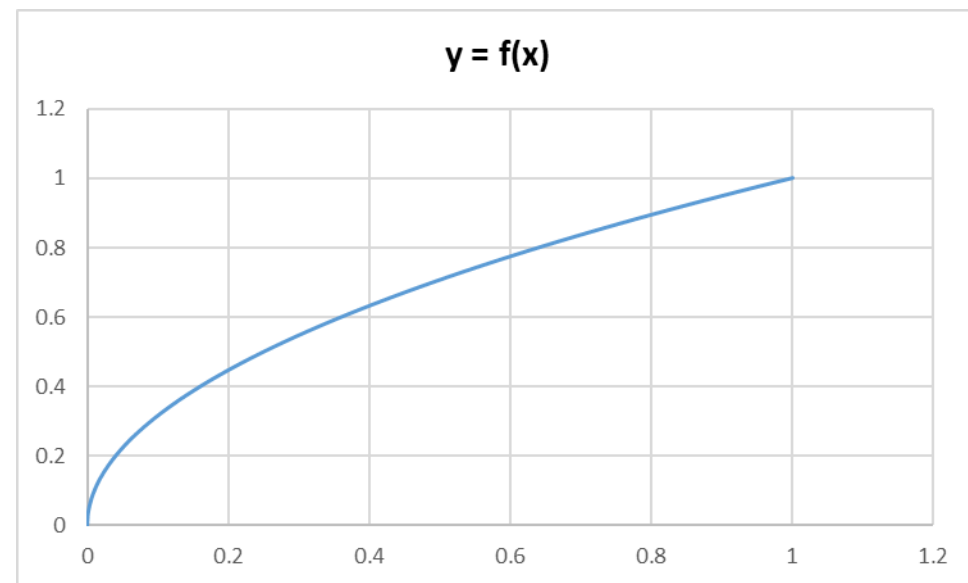


	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		A	b	c	d	n						
3		1	1	0	0	2						
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		0	0.001001	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	0	0				0	0	0	0.667161	0.667161	
12	2	0.001001	0.031639	0.001001	0.031639	31.60696	0.031639	3.17E-05	3.17E-05	-----	-----	0.031639
13	3	0.002002	0.044744	0.001001	0.013105	13.09203	0.044744	4.48E-05	7.65E-05	-----	-----	0.044744
14	4	0.003003	0.0548	0.001001	0.010056	10.04587	0.0548	5.49E-05	0.000131	-----	-----	0.0548
15	5	0.004004	0.063277	0.001001	0.008478	8.46906	0.063277	6.33E-05	0.000195	-----	-----	0.063277
16	6	0.005005	0.070746	0.001001	0.007469	7.461391	0.070746	7.08E-05	0.000265	-----	-----	0.070746
17	7	0.006006	0.077498	0.001001	0.006752	6.745613	0.077498	7.76E-05	0.000343	-----	-----	0.077498
18	8	0.007007	0.083708	0.001001	0.006209	6.203232	0.083708	8.38E-05	0.000427	-----	-----	0.083708
19	9	0.008008	0.089487	0.001001	0.00578	5.773827	0.089487	8.96E-05	0.000516	-----	-----	0.089487
20	10	0.009009	0.094916	0.001001	0.005428	5.422897	0.094916	9.5E-05	0.000611	-----	-----	0.094916
21	11	0.01001	0.10005	0.001001	0.005134	5.129104	0.10005	0.0001	0.000712	-----	-----	0.10005
22	12	0.011011	0.104933	0.001001	0.004883	4.878444	0.104933	0.000105	0.000817	-----	-----	0.104933
23	13	0.012012	0.109599	0.001001	0.004666	4.661294	0.109599	0.00011	0.000926	-----	-----	0.109599
24	14	0.013013	0.114075	0.001001	0.004475	4.470794	0.114075	0.000114	0.001041	-----	-----	0.114075
25	15	0.014014	0.118381	0.001001	0.004306	4.301901	0.118381	0.000118	0.001159	-----	-----	0.118381
26	16	0.015015	0.122536	0.001001	0.004155	4.150815	0.122536	0.000123	0.001282	-----	-----	0.122536
27	17	0.016016	0.126554	0.001001	0.004019	4.01461	0.126554	0.000127	0.001408	-----	-----	0.126554
28	18	0.017017	0.130449	0.001001	0.003895	3.890995	0.130449	0.000131	0.001539	-----	-----	0.130449
29	19	0.018018	0.134231	0.001001	0.003782	3.77814	0.134231	0.000134	0.001673	-----	-----	0.134231
30	20	0.019019	0.137909	0.001001	0.003678	3.67457	0.137909	0.000138	0.001811	-----	-----	0.137909

$$y = f(x) = A\sqrt[n]{b(x - c)} + d$$

$$y = f(x) = A[b(x - c)]^{1/n} + d$$

$$\int_0^1 \sqrt{x} dx = \frac{2}{3} \approx 0.666 \dots$$

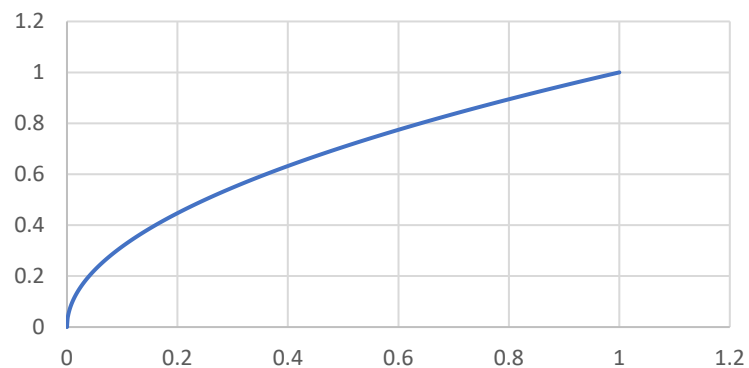


***n*th root functions**

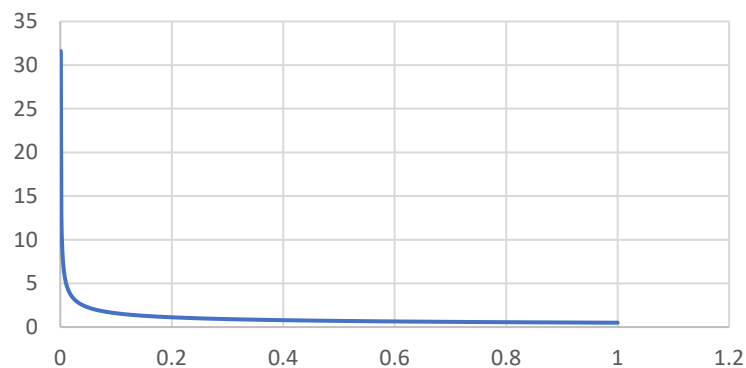
1007	997	0.996997	0.998497	0.001001	0.000501	0.500878	0.998497	0.000999	0.664159	-----	-----	0.998497
1008	998	0.997998	0.998998	0.001001	0.000501	0.500627	0.998998	0.001	0.665159	-----	-----	0.998998
1009	999	0.998999	0.999499	0.001001	0.000501	0.500376	0.999499	0.001	0.66616	-----	-----	0.999499
1010	1000	1	1	0.001001	0.000501	0.500125	1	0.001001	0.667161	-----	-----	1

$$f(x) = \sqrt{x}$$

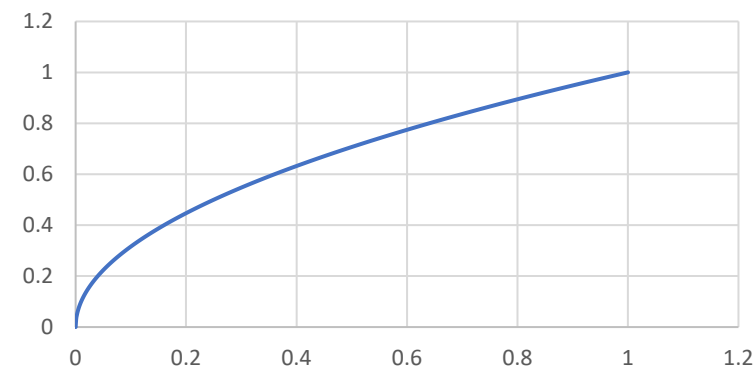
y = f(x)



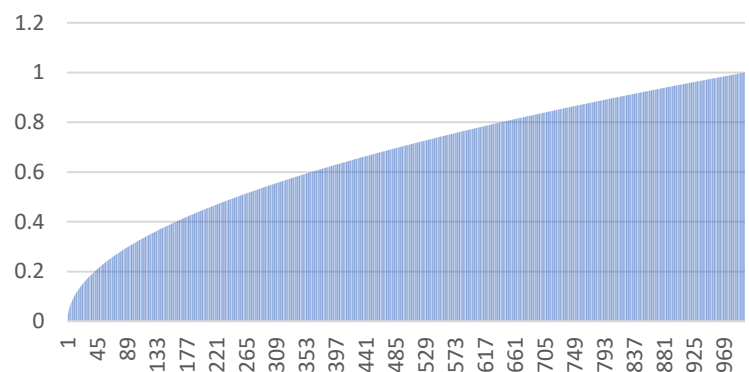
dy/dx



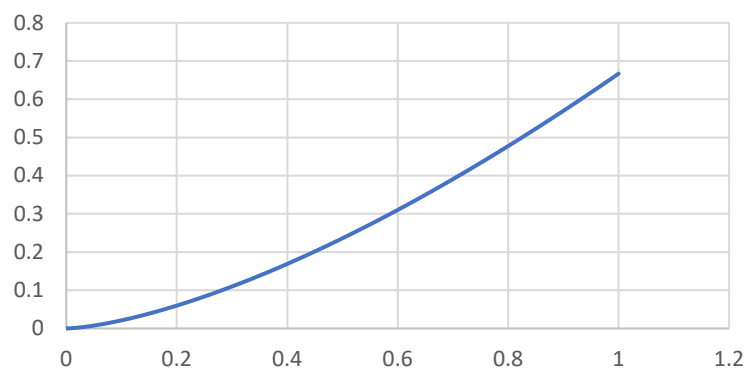
∫ dy = f(x)



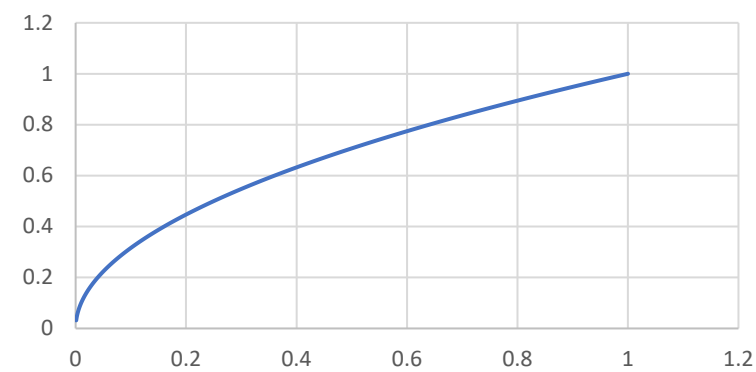
Rectangles of ∫ dy = f(x)



∫ f(x) dx

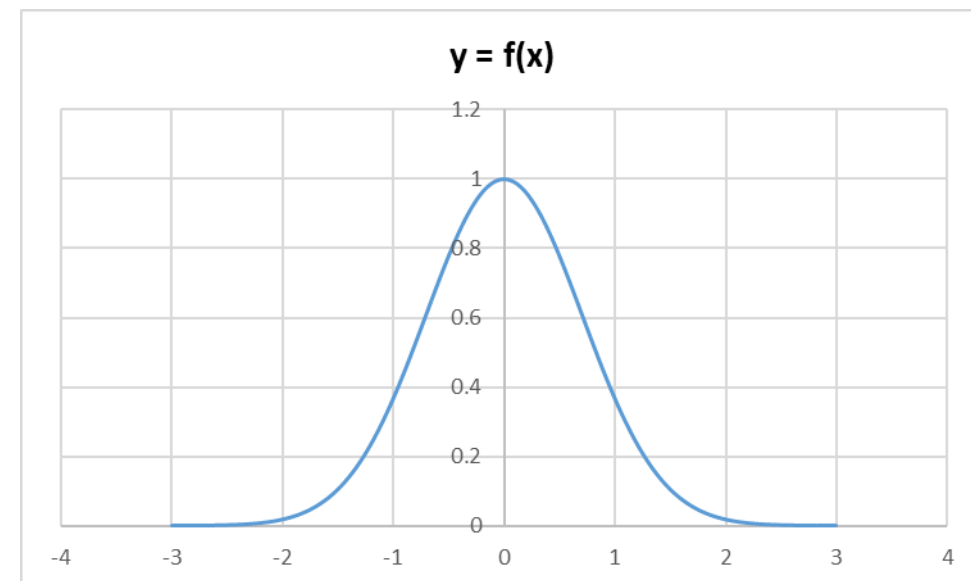


d/dx ∫ f(x) dx = f(x)



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		A	b	c	d		$y = f(x) = Ae^{-b(x-c)^2} + d$					
3		1	1	0	0							
4												
5			n							sqrt(pi)=	1.772454	
6			1000									
7		x min	dx	x max			Height	Area of				
8		-3	0.006006	3			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	-3	0.000123				0.000123	0	0	1.772415	1.772415	
12	2	-2.99399	0.000128	0.006006	4.52E-06	0.000753	0.000128	7.68E-07	7.68E-07	-----	-----	0.000128
13	3	-2.98799	0.000133	0.006006	4.68E-06	0.000779	0.000133	7.96E-07	1.56E-06	-----	-----	0.000133
14	4	-2.98198	0.000137	0.006006	4.84E-06	0.000806	0.000137	8.26E-07	2.39E-06	-----	-----	0.000137
15	5	-2.97598	0.000142	0.006006	5.01E-06	0.000834	0.000142	8.56E-07	3.25E-06	-----	-----	0.000142
16	6	-2.96997	0.000148	0.006006	5.18E-06	0.000862	0.000148	8.87E-07	4.13E-06	-----	-----	0.000148
17	7	-2.96396	0.000153	0.006006	5.36E-06	0.000892	0.000153	9.19E-07	5.05E-06	-----	-----	0.000153
18	8	-2.95796	0.000159	0.006006	5.54E-06	0.000922	0.000159	9.52E-07	6E-06	-----	-----	0.000159
19	9	-2.95195	0.000164	0.006006	5.73E-06	0.000954	0.000164	9.87E-07	6.99E-06	-----	-----	0.000164
20	10	-2.94595	0.00017	0.006006	5.92E-06	0.000986	0.00017	1.02E-06	8.01E-06	-----	-----	0.00017
21	11	-2.93994	0.000176	0.006006	6.12E-06	0.00102	0.000176	1.06E-06	9.07E-06	-----	-----	0.000176
22	12	-2.93393	0.000183	0.006006	6.33E-06	0.001054	0.000183	1.1E-06	1.02E-05	-----	-----	0.000183
23	13	-2.92793	0.000189	0.006006	6.54E-06	0.00109	0.000189	1.14E-06	1.13E-05	-----	-----	0.000189
24	14	-2.92192	0.000196	0.006006	6.77E-06	0.001126	0.000196	1.18E-06	1.25E-05	-----	-----	0.000196
25	15	-2.91592	0.000203	0.006006	6.99E-06	0.001164	0.000203	1.22E-06	1.37E-05	-----	-----	0.000203
26	16	-2.90991	0.00021	0.006006	7.23E-06	0.001203	0.00021	1.26E-06	1.5E-05	-----	-----	0.00021
27	17	-2.9039	0.000218	0.006006	7.47E-06	0.001243	0.000218	1.31E-06	1.63E-05	-----	-----	0.000218
28	18	-2.8979	0.000225	0.006006	7.72E-06	0.001285	0.000225	1.35E-06	1.76E-05	-----	-----	0.000225
29	19	-2.89189	0.000233	0.006006	7.97E-06	0.001328	0.000233	1.4E-06	1.9E-05	-----	-----	0.000233
30	20	-2.88589	0.000242	0.006006	8.24E-06	0.001372	0.000242	1.45E-06	2.05E-05	-----	-----	0.000242

$$\int_{-3}^3 e^{-x^2} dx = \sqrt{\pi} \approx 1.77245 \dots$$

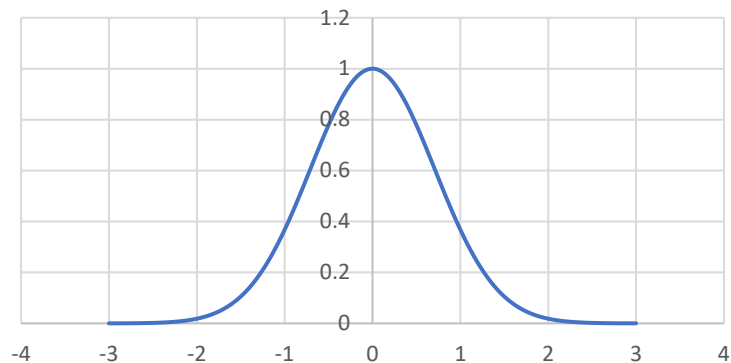


Gaussian integral

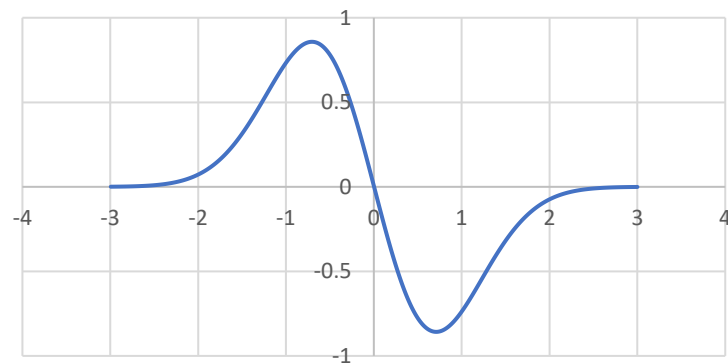
1007	997	2.981982	0.000137	0.006006	-5E-06	-0.00083	0.000137	8.26E-07	1.772412	-----	-----	0.000137
1008	998	2.987988	0.000133	0.006006	-4.8E-06	-0.00081	0.000133	7.96E-07	1.772413	-----	-----	0.000133
1009	999	2.993994	0.000128	0.006006	-4.7E-06	-0.00078	0.000128	7.68E-07	1.772414	-----	-----	0.000128
1010	1000	3	0.000123	0.006006	-4.5E-06	-0.00075	0.000123	7.41E-07	1.772415	-----	-----	0.000123

$$f(x) = e^{-x^2}$$

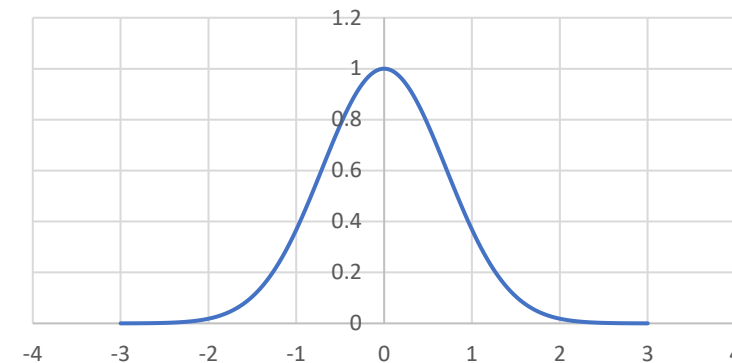
y = f(x)



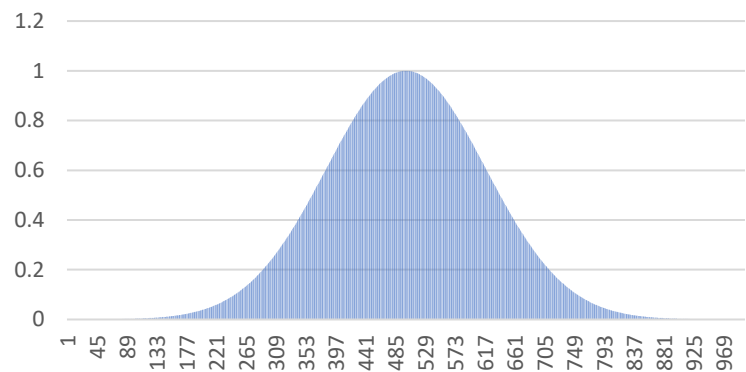
dy/dx



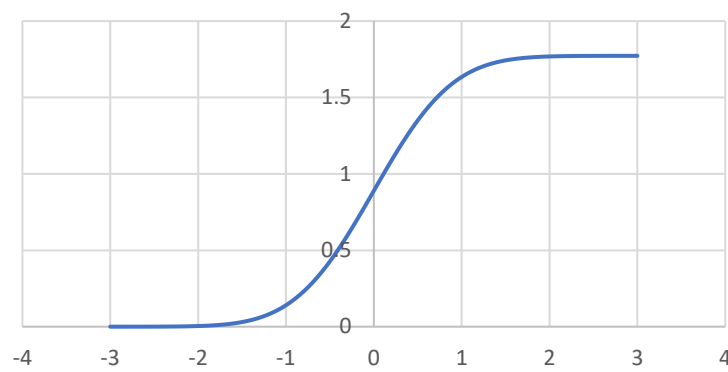
$\int dy = f(x)$



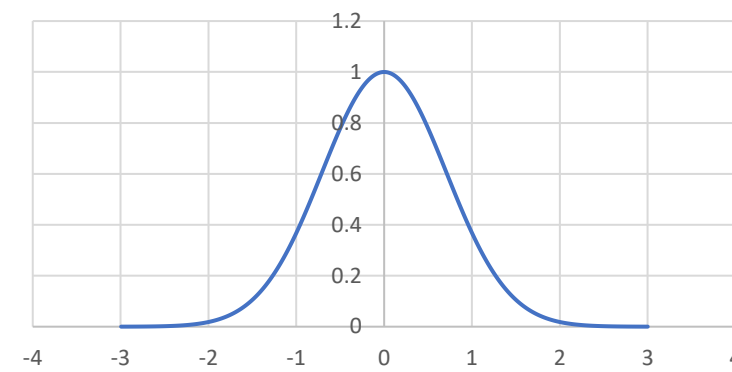
Rectangles of $\int dy = f(x)$



$\int f(x) dx$



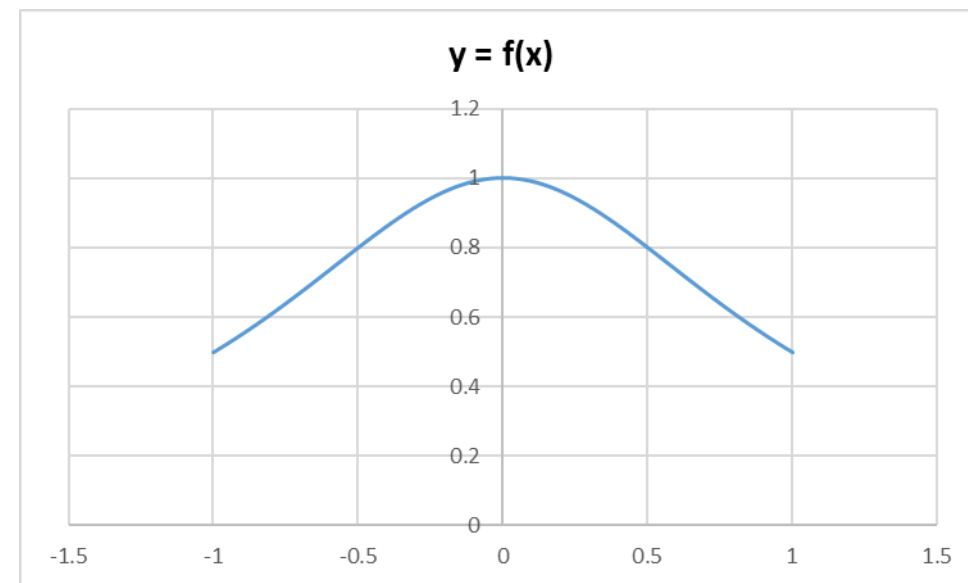
$d/dx \int f(x) dx = f(x)$



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2												
3												
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		-1	0.002002	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	-1	0.5				0.5	0	0	1.570796	1.570796	
12	2	-0.998	0.501002	0.002002	0.001002	0.5005	0.501002	0.001003	0.001003	-----	-----	0.501002
13	3	-0.996	0.502006	0.002002	0.001004	0.501501	0.502006	0.001005	0.002008	-----	-----	0.502006
14	4	-0.99399	0.503012	0.002002	0.001006	0.502502	0.503012	0.001007	0.003015	-----	-----	0.503012
15	5	-0.99199	0.50402	0.002002	0.001008	0.503503	0.50402	0.001009	0.004024	-----	-----	0.50402
16	6	-0.98999	0.50503	0.002002	0.00101	0.504504	0.50503	0.001011	0.005035	-----	-----	0.50503
17	7	-0.98799	0.506042	0.002002	0.001012	0.505505	0.506042	0.001013	0.006048	-----	-----	0.506042
18	8	-0.98599	0.507056	0.002002	0.001014	0.506505	0.507056	0.001015	0.007063	-----	-----	0.507056
19	9	-0.98398	0.508072	0.002002	0.001016	0.507506	0.508072	0.001017	0.008081	-----	-----	0.508072
20	10	-0.98198	0.50909	0.002002	0.001018	0.508506	0.50909	0.001019	0.0091	-----	-----	0.50909
21	11	-0.97998	0.51011	0.002002	0.00102	0.509506	0.51011	0.001021	0.010121	-----	-----	0.51011
22	12	-0.97798	0.511132	0.002002	0.001022	0.510506	0.511132	0.001023	0.011144	-----	-----	0.511132
23	13	-0.97598	0.512156	0.002002	0.001024	0.511505	0.512156	0.001025	0.01217	-----	-----	0.512156
24	14	-0.97397	0.513182	0.002002	0.001026	0.512504	0.513182	0.001027	0.013197	-----	-----	0.513182
25	15	-0.97197	0.51421	0.002002	0.001028	0.513503	0.51421	0.001029	0.014226	-----	-----	0.51421
26	16	-0.96997	0.51524	0.002002	0.00103	0.514502	0.51524	0.001032	0.015258	-----	-----	0.51524
27	17	-0.96797	0.516272	0.002002	0.001032	0.5155	0.516272	0.001034	0.016292	-----	-----	0.516272
28	18	-0.96597	0.517306	0.002002	0.001034	0.516498	0.517306	0.001036	0.017327	-----	-----	0.517306
29	19	-0.96396	0.518342	0.002002	0.001036	0.517495	0.518342	0.001038	0.018365	-----	-----	0.518342
30	20	-0.96196	0.51938	0.002002	0.001038	0.518492	0.51938	0.00104	0.019405	-----	-----	0.51938

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{x^2 + 1}$$

$$\int_{-1}^1 \frac{1}{x^2 + 1} dx = \frac{\pi}{2} = 1.5707 \dots$$

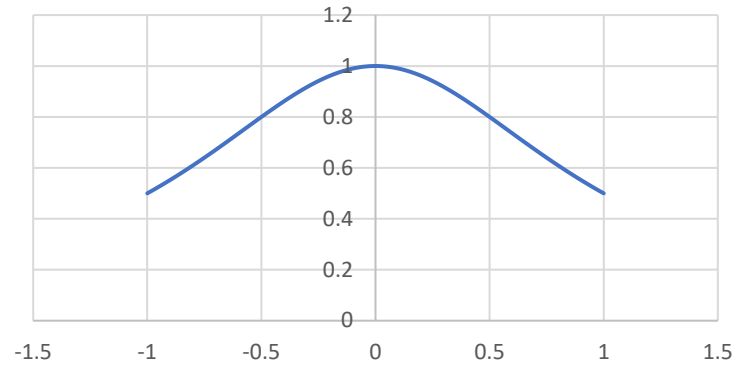


Derivative of the arctangent function

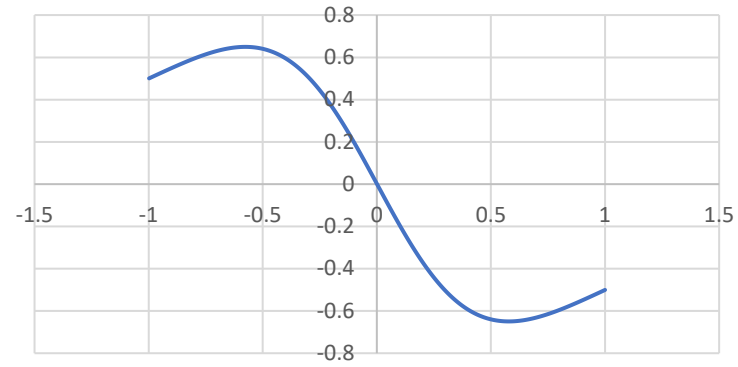
1007	997	0.993994	0.503012	0.002002	-0.00101	-0.5035	0.503012	0.001007	1.567787	-----	-----	0.503012
1008	998	0.995996	0.502006	0.002002	-0.00101	-0.5025	0.502006	0.001005	1.568792	-----	-----	0.502006
1009	999	0.997998	0.501002	0.002002	-0.001	-0.5015	0.501002	0.001003	1.569795	-----	-----	0.501002
1010	1000	1	0.5	0.002002	-0.001	-0.5005	0.5	0.001001	1.570796	-----	-----	0.5

$$f(x) = \frac{1}{x^2 + 1}$$

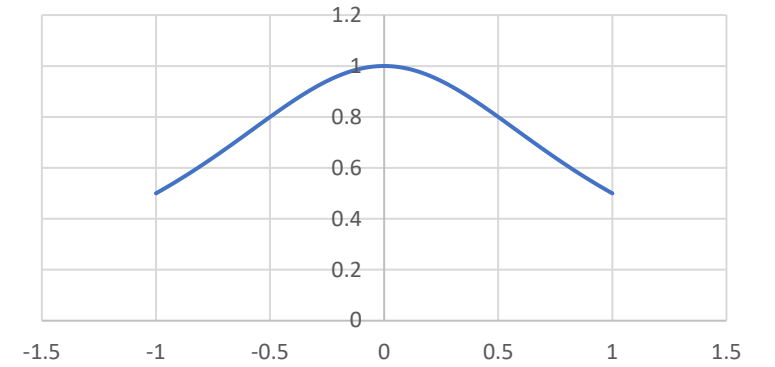
y = f(x)



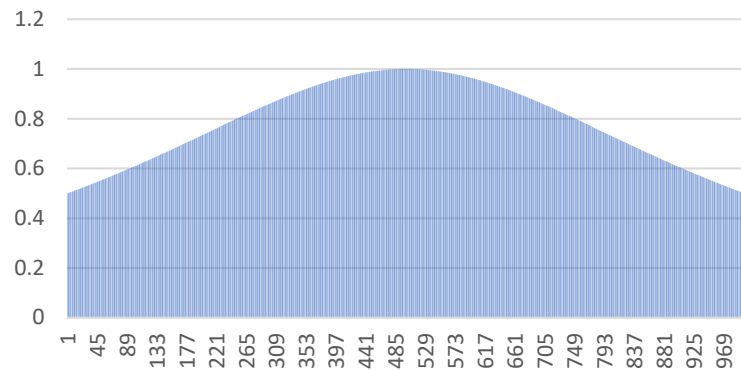
dy/dx



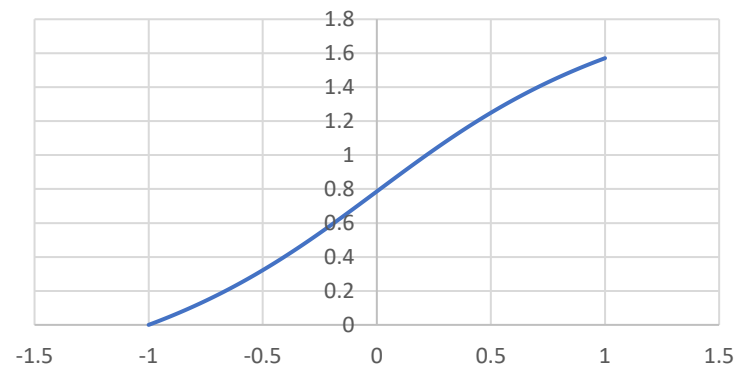
$\int dy = f(x)$



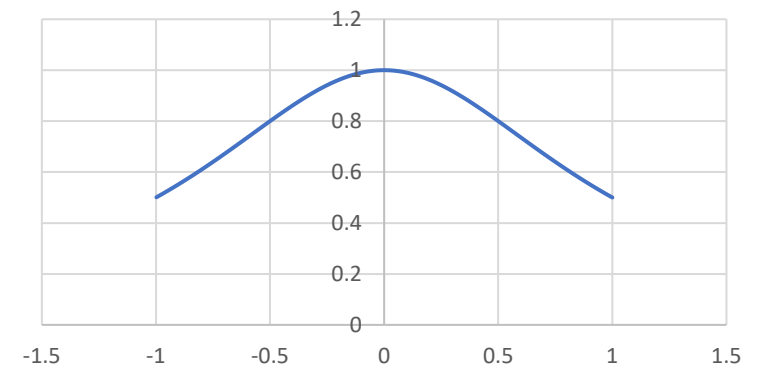
Rectangles of $\int dy = f(x)$



$\int f(x) dx$



d/dx $\int f(x) dx$

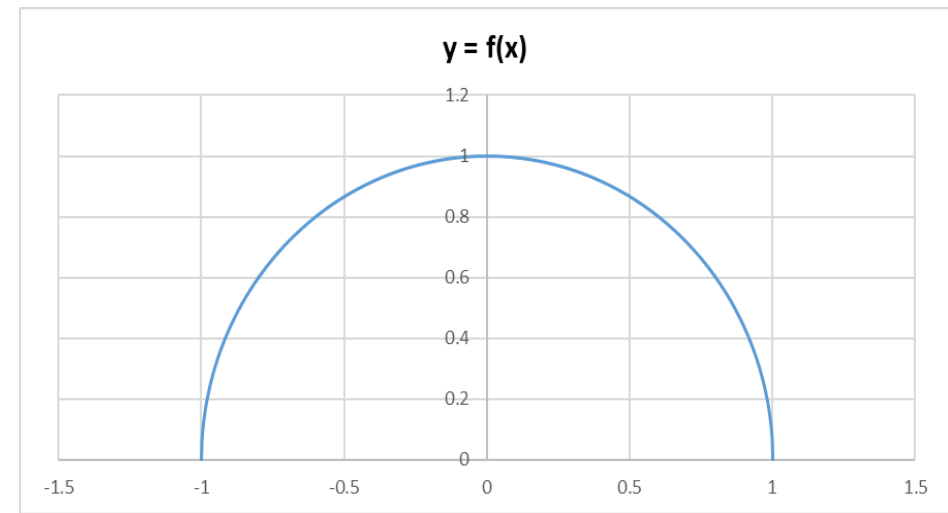


	A	B	C	D	E	F	G	H	I	J	K	L
1												
2			r									
3			1									
4												
5			n									
6			1000									
7		x min	dx	x max			Height	Area of				
8		-1	0.002002	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	-1	0				0	0	0	1.570744	1.570744	
12	2	-0.998	0.063246	0.002002	0.063246	31.59114	0.063246	0.000127	0.000127	-----	-----	0.063246
13	3	-0.996	0.089398	0.002002	0.026152	13.06309	0.089398	0.000179	0.000306	-----	-----	0.089398
14	4	-0.99399	0.109435	0.002002	0.020037	10.00837	0.109435	0.000219	0.000525	-----	-----	0.109435
15	5	-0.99199	0.126301	0.002002	0.016866	8.42464	0.126301	0.000253	0.000778	-----	-----	0.126301
16	6	-0.98999	0.141138	0.002002	0.014837	7.410986	0.141138	0.000283	0.00106	-----	-----	0.141138
17	7	-0.98799	0.154531	0.002002	0.013393	6.689855	0.154531	0.000309	0.001369	-----	-----	0.154531
18	8	-0.98599	0.166828	0.002002	0.012297	6.142585	0.166828	0.000334	0.001703	-----	-----	0.166828
19	9	-0.98398	0.178257	0.002002	0.011429	5.708651	0.178257	0.000357	0.00206	-----	-----	0.178257
20	10	-0.98198	0.188975	0.002002	0.010718	5.353479	0.188975	0.000378	0.002439	-----	-----	0.188975
21	11	-0.97998	0.199096	0.002002	0.010121	5.055683	0.199096	0.000399	0.002837	-----	-----	0.199096
22	12	-0.97798	0.208708	0.002002	0.009612	4.801221	0.208708	0.000418	0.003255	-----	-----	0.208708
23	13	-0.97598	0.217878	0.002002	0.00917	4.580443	0.217878	0.000436	0.003691	-----	-----	0.217878
24	14	-0.97397	0.22666	0.002002	0.008782	4.386464	0.22666	0.000454	0.004145	-----	-----	0.22666
25	15	-0.97197	0.235097	0.002002	0.008437	4.214224	0.235097	0.000471	0.004616	-----	-----	0.235097
26	16	-0.96997	0.243225	0.002002	0.008128	4.05991	0.243225	0.000487	0.005103	-----	-----	0.243225
27	17	-0.96797	0.251074	0.002002	0.007849	3.920583	0.251074	0.000503	0.005605	-----	-----	0.251074
28	18	-0.96597	0.258669	0.002002	0.007595	3.79394	0.258669	0.000518	0.006123	-----	-----	0.258669
29	19	-0.96396	0.266033	0.002002	0.007364	3.678144	0.266033	0.000533	0.006656	-----	-----	0.266033
30	20	-0.96196	0.273183	0.002002	0.007151	3.571713	0.273183	0.000547	0.007203	-----	-----	0.273183

$$y = f(x) = \sqrt{r^2 - x^2}$$

Area of a circle = πr^2

$$\int_{-1}^1 \sqrt{1 - x^2} dx = \frac{\pi}{2} = 1.5707 \dots$$

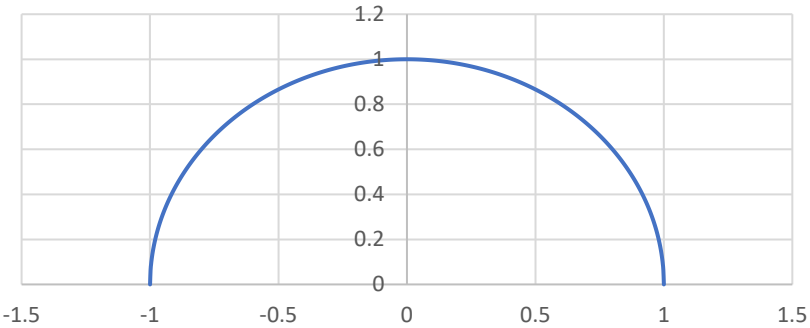


Circle

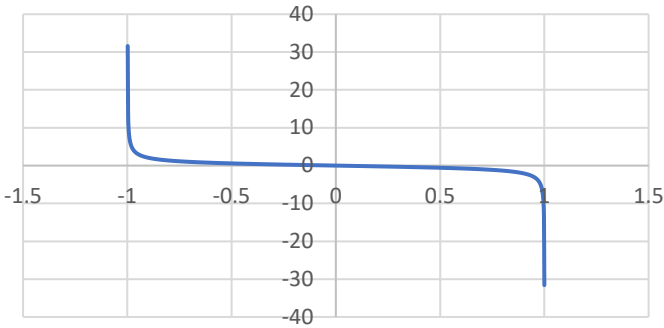
1007	997	0.993994	0.109435	0.002002	-0.01687	-8.42464	0.109435	0.000219	1.570438	-----	-----	0.109435
1008	998	0.995996	0.089398	0.002002	-0.02004	-10.0084	0.089398	0.000179	1.570617	-----	-----	0.089398
1009	999	0.997998	0.063246	0.002002	-0.02615	-13.0631	0.063246	0.000127	1.570744	-----	-----	0.063246
1010	1000	1	0	0.002002	-0.06325	-31.5911	0	0	1.570744	-----	-----	0

$$f(x) = \sqrt{1 - x^2}$$

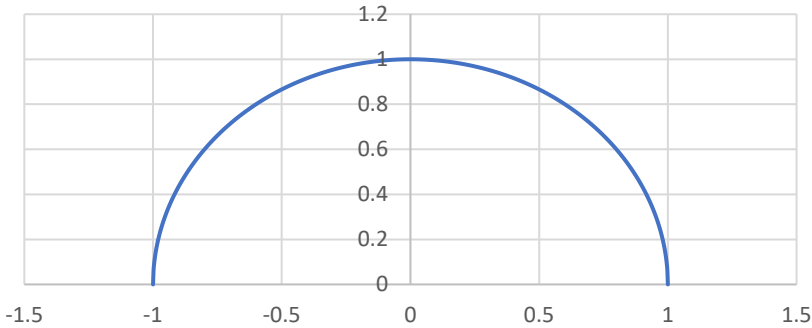
y = f(x)



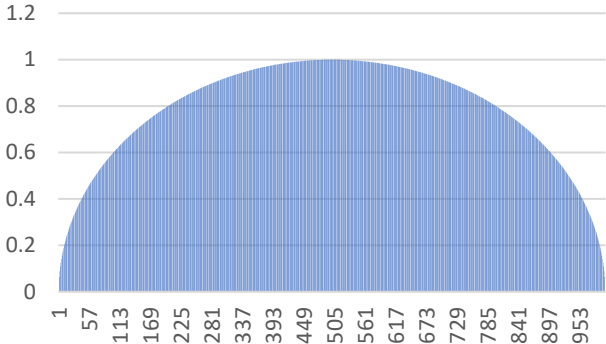
dy/dx



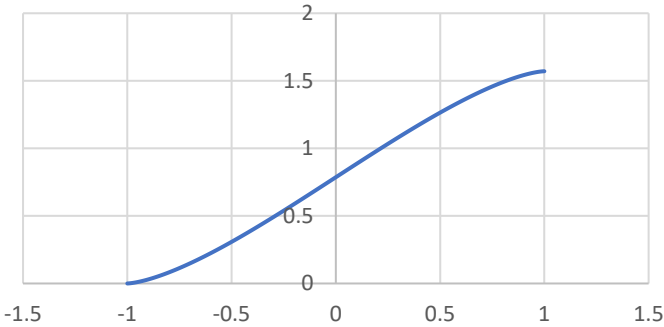
∫ dy = f(x)



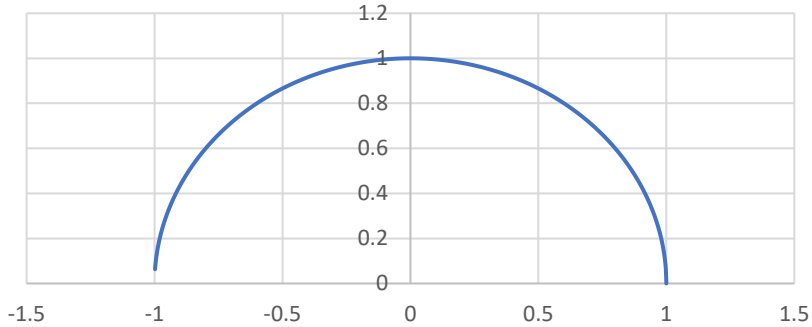
Rectangles of ∫ dy = f(x)



∫ f(x) dx



d/dx ∫ f(x) dx = f(x)

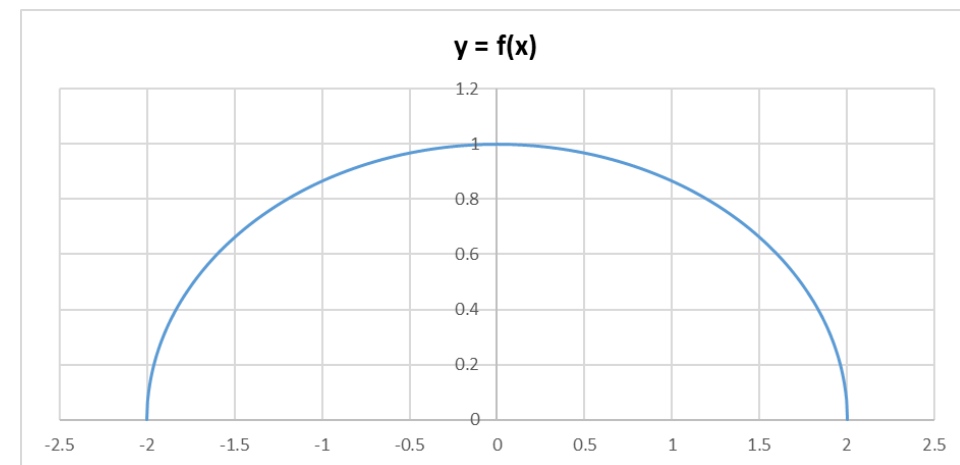


	A	B	C	D	E	F	G	H	I	J	K	L
1												
2					semi-	semi-						
3					major	major						
4					axis (a)	axis (b)						
5			n		1	2						
6			1000									
7		x min	dx	x max			Height	Area of				
8		-1	0.002002	1			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	-1	0				0	0	0	3.141487	3.141487	
12	2	-0.998	0.126491	0.002002	0.126491	63.18228	0.126491	0.000253	0.000253	-----	-----	0.126491
13	3	-0.996	0.178796	0.002002	0.052305	26.12618	0.178796	0.000358	0.000611	-----	-----	0.178796
14	4	-0.99399	0.218869	0.002002	0.040074	20.01675	0.218869	0.000438	0.001049	-----	-----	0.218869
15	5	-0.99199	0.252602	0.002002	0.033732	16.84928	0.252602	0.000506	0.001555	-----	-----	0.252602
16	6	-0.98999	0.282275	0.002002	0.029674	14.82197	0.282275	0.000565	0.00212	-----	-----	0.282275
17	7	-0.98799	0.309061	0.002002	0.026786	13.37971	0.309061	0.000619	0.002739	-----	-----	0.309061
18	8	-0.98599	0.333656	0.002002	0.024595	12.28517	0.333656	0.000668	0.003407	-----	-----	0.333656
19	9	-0.98398	0.356514	0.002002	0.022857	11.4173	0.356514	0.000714	0.004121	-----	-----	0.356514
20	10	-0.98198	0.377949	0.002002	0.021435	10.70696	0.377949	0.000757	0.004877	-----	-----	0.377949
21	11	-0.97998	0.398192	0.002002	0.020243	10.11137	0.398192	0.000797	0.005674	-----	-----	0.398192
22	12	-0.97798	0.417416	0.002002	0.019224	9.602442	0.417416	0.000836	0.00651	-----	-----	0.417416
23	13	-0.97598	0.435756	0.002002	0.01834	9.160885	0.435756	0.000872	0.007383	-----	-----	0.435756
24	14	-0.97397	0.45332	0.002002	0.017563	8.772927	0.45332	0.000908	0.00829	-----	-----	0.45332
25	15	-0.97197	0.470194	0.002002	0.016874	8.428448	0.470194	0.000941	0.009231	-----	-----	0.470194
26	16	-0.96997	0.486449	0.002002	0.016256	8.119819	0.486449	0.000974	0.010205	-----	-----	0.486449
27	17	-0.96797	0.502147	0.002002	0.015698	7.841166	0.502147	0.001005	0.011211	-----	-----	0.502147
28	18	-0.96597	0.517338	0.002002	0.015191	7.58788	0.517338	0.001036	0.012246	-----	-----	0.517338
29	19	-0.96396	0.532066	0.002002	0.014727	7.356289	0.532066	0.001065	0.013311	-----	-----	0.532066
30	20	-0.96196	0.546367	0.002002	0.014301	7.143427	0.546367	0.001094	0.014405	-----	-----	0.546367



1007	997	0.993994	0.218869	0.002002	-0.03373	-16.8493	0.218869	0.000438	3.140876	-----	-----	0.218869
1008	998	0.995996	0.178796	0.002002	-0.04007	-20.0167	0.178796	0.000358	3.141234	-----	-----	0.178796
1009	999	0.997998	0.126491	0.002002	-0.0523	-26.1262	0.126491	0.000253	3.141487	-----	-----	0.126491
1010	1000	1	0	0.002002	-0.12649	-63.1823	0	0	3.141487	-----	-----	0

$$\int_{-1}^1 \sqrt{2^2 \left(1 - \frac{x^2}{1^2}\right)} dx = \pi = 3.14159 \dots$$

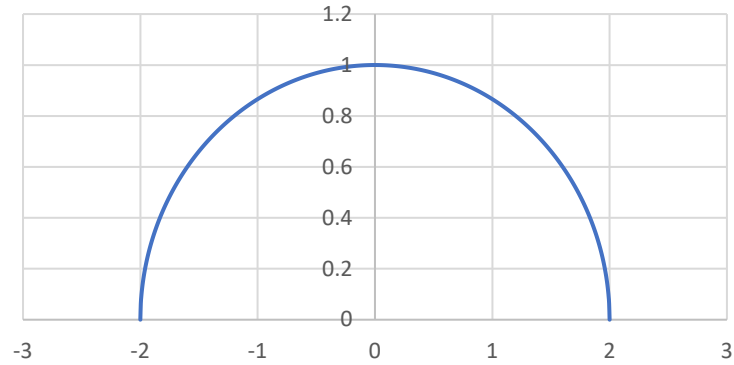


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y = f(x) = \sqrt{b^2 \left(1 - \frac{x^2}{a^2}\right)}$$

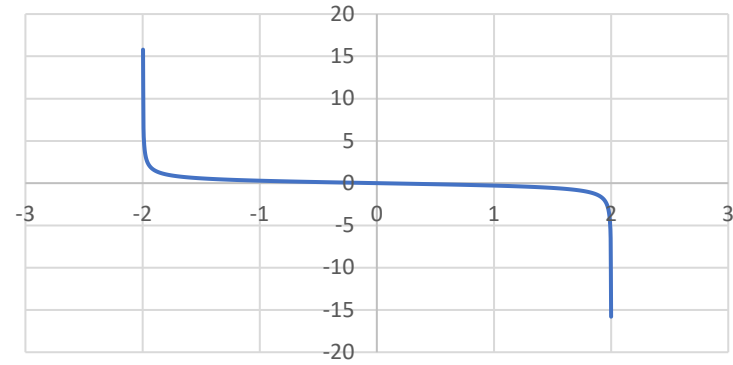
Ellipse

$$f(x) = \sqrt{b^2 \left(1 - \frac{x^2}{a^2}\right)}$$

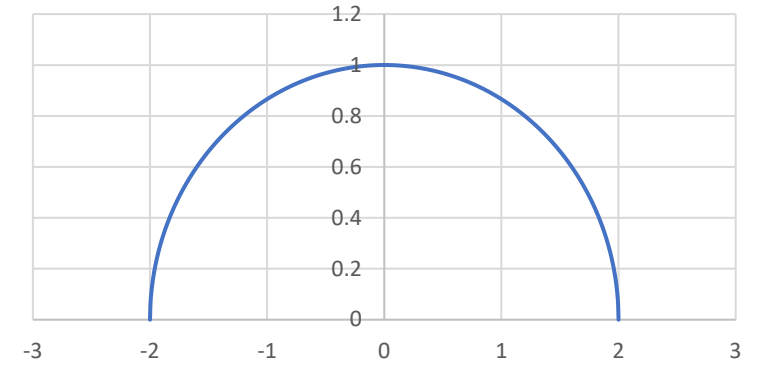
y = f(x)



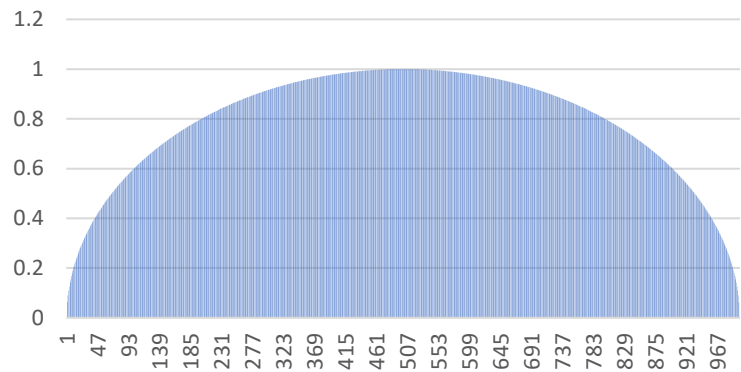
dy/dx



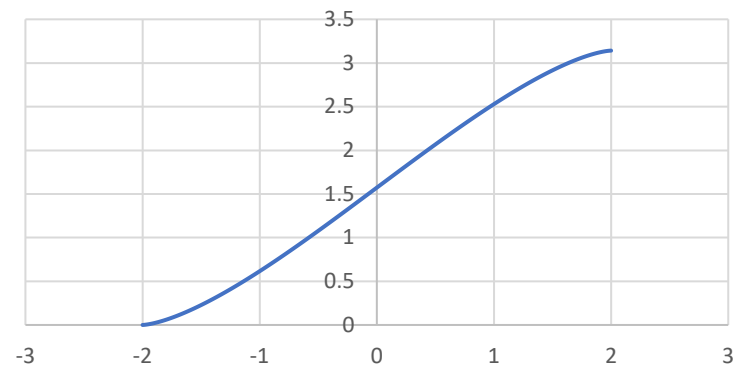
∫ dy = f(x)



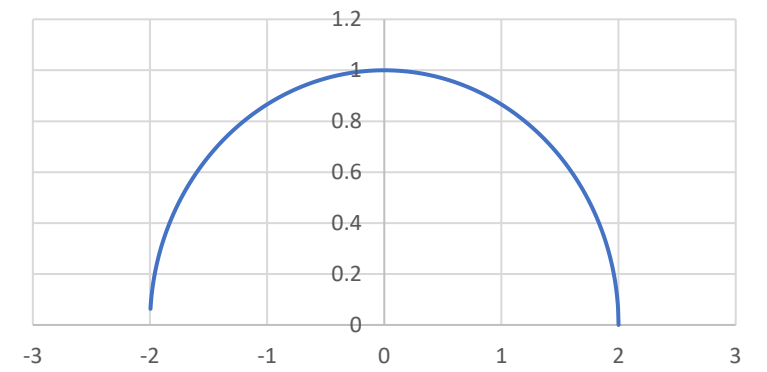
Rectangles of ∫ dy = f(x)



∫ f(x) dx

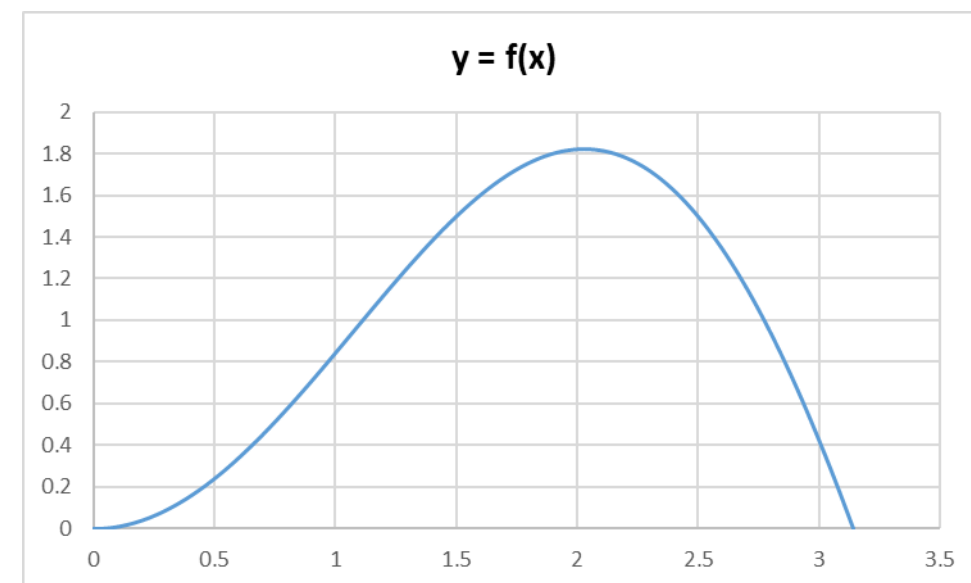


d/dx ∫ f(x) dx = f(x)



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2												
3												
4												
5		k	n	k								
6		0	1000	1								
7		x min	dx	x max			Height	Area of				
8		0	0.003145	3.141593			of each	each	Cumulative			f(x) =
9							rectangle	rectangle	area	Total	Final	d/dx of
10	n	x	y = f(x)	dx	dy	dy / dx	∫ dy = f(x)	f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx	∫ f(x) dx
11	1	0	0				0	0	0	3.14159	3.14159	
12	2	0.003145	9.89E-06	0.003145	9.89E-06	0.003145	9.89E-06	3.11E-08	3.11E-08	-----	-----	9.89E-06
13	3	0.006289	3.96E-05	0.003145	2.97E-05	0.009434	3.96E-05	1.24E-07	1.55E-07	-----	-----	3.96E-05
14	4	0.009434	8.9E-05	0.003145	4.94E-05	0.015723	8.9E-05	2.8E-07	4.35E-07	-----	-----	8.9E-05
15	5	0.012579	0.000158	0.003145	6.92E-05	0.022012	0.000158	4.98E-07	9.33E-07	-----	-----	0.000158
16	6	0.015724	0.000247	0.003145	8.9E-05	0.028301	0.000247	7.77E-07	1.71E-06	-----	-----	0.000247
17	7	0.018868	0.000356	0.003145	0.000109	0.034589	0.000356	1.12E-06	2.83E-06	-----	-----	0.000356
18	8	0.022013	0.000485	0.003145	0.000129	0.040876	0.000485	1.52E-06	4.35E-06	-----	-----	0.000485
19	9	0.025158	0.000633	0.003145	0.000148	0.047162	0.000633	1.99E-06	6.34E-06	-----	-----	0.000633
20	10	0.028303	0.000801	0.003145	0.000168	0.053448	0.000801	2.52E-06	8.86E-06	-----	-----	0.000801
21	11	0.031447	0.000989	0.003145	0.000188	0.059732	0.000989	3.11E-06	1.2E-05	-----	-----	0.000989
22	12	0.034592	0.001196	0.003145	0.000208	0.066015	0.001196	3.76E-06	1.57E-05	-----	-----	0.001196
23	13	0.037737	0.001424	0.003145	0.000227	0.072297	0.001424	4.48E-06	2.02E-05	-----	-----	0.001424
24	14	0.040882	0.001671	0.003145	0.000247	0.078578	0.001671	5.25E-06	2.55E-05	-----	-----	0.001671
25	15	0.044026	0.001938	0.003145	0.000267	0.084857	0.001938	6.09E-06	3.16E-05	-----	-----	0.001938
26	16	0.047171	0.002224	0.003145	0.000287	0.091134	0.002224	6.99E-06	3.86E-05	-----	-----	0.002224
27	17	0.050316	0.002531	0.003145	0.000306	0.09741	0.002531	7.96E-06	4.65E-05	-----	-----	0.002531
28	18	0.053461	0.002857	0.003145	0.000326	0.103683	0.002857	8.98E-06	5.55E-05	-----	-----	0.002857
29	19	0.056605	0.003202	0.003145	0.000346	0.109955	0.003202	1.01E-05	6.56E-05	-----	-----	0.003202
30	20	0.05975	0.003568	0.003145	0.000365	0.116224	0.003568	1.12E-05	7.68E-05	-----	-----	0.003568

$$\int_0^{\pi} x \sin(x) dx = \pi = 3.14159 \dots$$



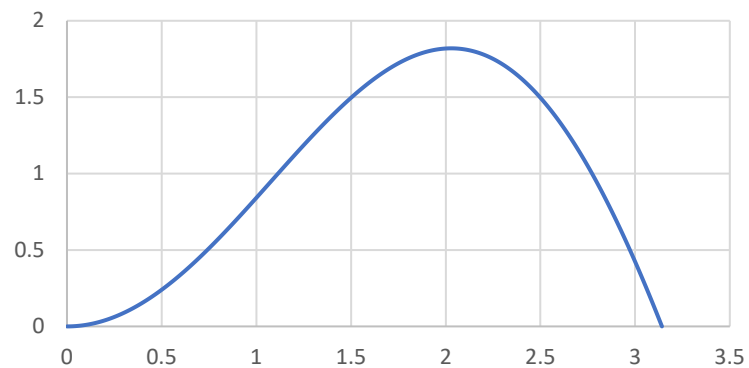
2024 VCAA Mathematical Methods Exam 2 Question 7

$$f(x) = x \sin(x)$$

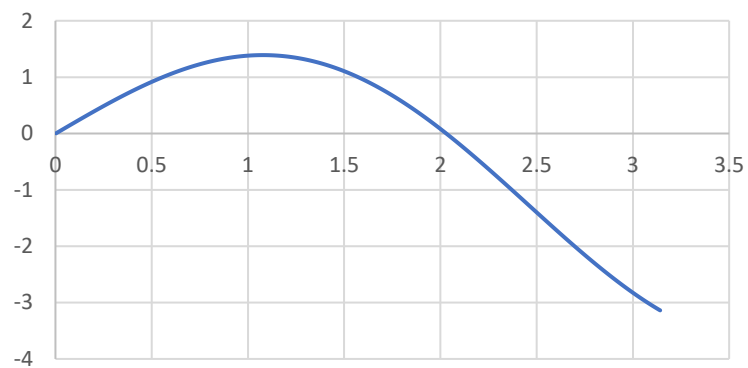
1007	997	3.132158	0.029549	0.003145	-0.00981	-3.11939	0.029549	9.29E-05	3.141497	-----	-----	0.029549
1008	998	3.135303	0.019719	0.003145	-0.00983	-3.12577	0.019719	6.2E-05	3.141559	-----	-----	0.019719
1009	999	3.138448	0.00987	0.003145	-0.00985	-3.13212	0.00987	3.1E-05	3.14159	-----	-----	0.00987
1010	1000	3.141593	3.85E-16	0.003145	-0.00987	-3.13844	3.85E-16	1.21E-18	3.14159	-----	-----	0

$$f(x) = x \sin(x)$$

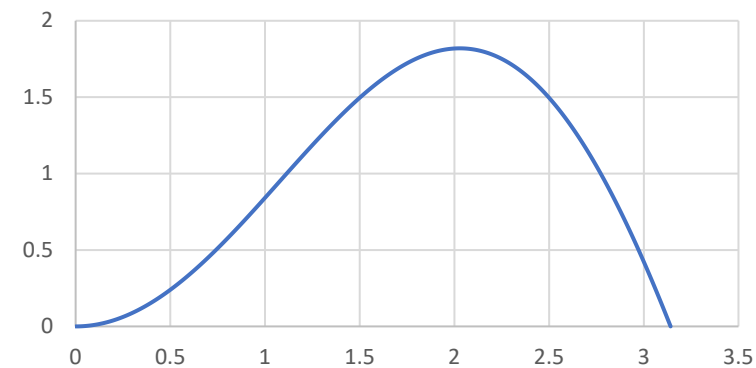
$y = f(x)$



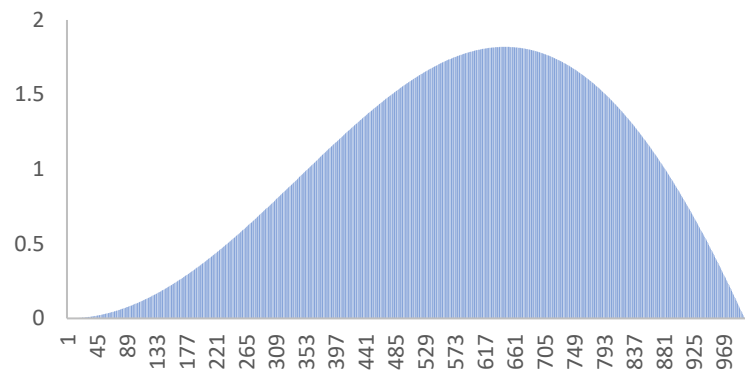
dy/dx



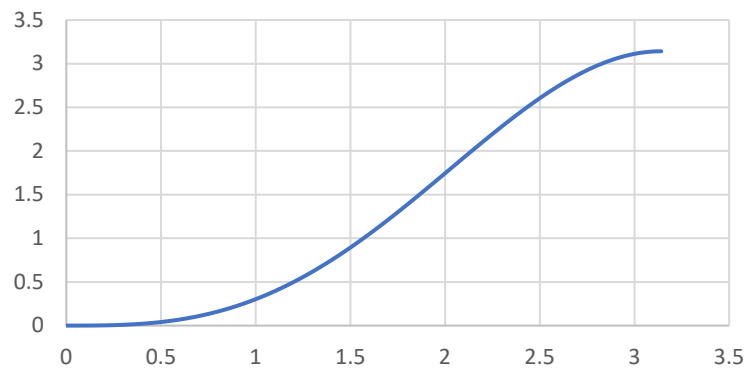
$\int dy = f(x)$



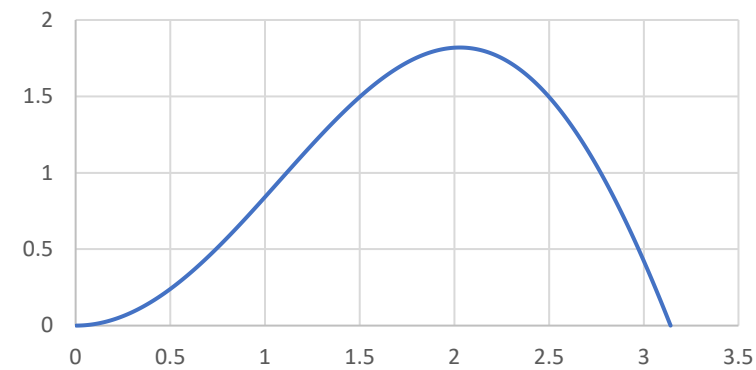
Rectangles of $\int dy = f(x)$



$\int f(x) dx$



$d/dx \int f(x) dx = f(x)$



Note the pattern:

$$\int_0^{\pi} x \sin(x) dx = \pi$$

$$\int_{\pi}^{2\pi} x \sin(x) dx = -3\pi$$

$$\int_{2\pi}^{3\pi} x \sin(x) dx = 5\pi$$

$$\int_{3\pi}^{4\pi} x \sin(x) dx = -7\pi$$

$$\int_{4\pi}^{5\pi} x \sin(x) dx = 9\pi$$

$$\int_{5\pi}^{6\pi} x \sin(x) dx = -11\pi$$

$$\int_{6\pi}^{7\pi} x \sin(x) dx = 13\pi$$

$$\int_{7\pi}^{8\pi} x \sin(x) dx = -15\pi$$

$$\int_{8\pi}^{9\pi} x \sin(x) dx = 17\pi$$

$$\int_{9\pi}^{10\pi} x \sin(x) dx = -19\pi$$

$$\int_{10\pi}^{11\pi} x \sin(x) dx = 21\pi$$

$$\int_{11\pi}^{12\pi} x \sin(x) dx = -23\pi$$

$$\int_{12\pi}^{13\pi} x \sin(x) dx = 25\pi$$

$$\int_{13\pi}^{14\pi} x \sin(x) dx = -27\pi$$

$$\int_{14\pi}^{15\pi} x \sin(x) dx = 29\pi \text{ and so on ...}$$

Calculate d/dx of $x \cdot \sin(x)$ showing all the steps.

Sure! Let's calculate the derivative of $x \sin(x)$ with respect to x using the product rule. The product rule states that if you have a function $u(x)$ and $v(x)$, then the derivative of their product is given by:

$$\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

In this case, let $u(x) = x$ and $v(x) = \sin(x)$.

1. Differentiate $u(x)$ and $v(x)$:

- $u(x) = x \implies u'(x) = 1$
- $v(x) = \sin(x) \implies v'(x) = \cos(x)$

2. Apply the product rule:

$$\frac{d}{dx}[x \sin(x)] = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

$$\frac{d}{dx}[x \sin(x)] = 1 \cdot \sin(x) + x \cdot \cos(x)$$

AI: Microsoft Copilot

3. Simplify the expression:

$$\frac{d}{dx}[x \sin(x)] = \sin(x) + x \cos(x)$$

So, the derivative of $x \sin(x)$ with respect to x is:

$$\frac{d}{dx}[x \sin(x)] = \sin(x) + x \cos(x)$$

Isn't it satisfying to see how the product rule works? If you have any more questions or need further clarification, feel free to ask!

Calculate the integral of $x \sin(x)$ dx showing all the steps.

To calculate the integral of $x \sin(x) dx$, we can use the technique of integration by parts. The formula for integration by parts is:

$$\int u dv = uv - \int v du$$

Let's choose u and dv as follows:

- $u = x \implies du = dx$
- $dv = \sin(x) dx \implies v = -\cos(x)$

Now, apply the integration by parts formula:

$$\int x \sin(x) dx = x(-\cos(x)) - \int (-\cos(x)) dx$$

Simplify the expression:

$$\int x \sin(x) dx = -x \cos(x) + \int \cos(x) dx$$

Now, integrate $\cos(x)$:

$$\int \cos(x) dx = \sin(x)$$

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So, we have:

$$\int x \sin(x) dx = -x \cos(x) + \sin(x) + C$$

where C is the constant of integration.

Therefore, the integral of $x \sin(x) dx$ is:

$$\int x \sin(x) dx = -x \cos(x) + \sin(x) + C$$

Isn't it satisfying to see how integration by parts works? If you have any more questions or need further clarification, feel free to ask!

Calculate the integral of $\sin(x) + x\cos(x)$ showing all the steps.

Sure! Let's calculate the integral of $\sin(x) + x\cos(x) dx$ step by step.

1. Separate the integral:

$$\int (\sin(x) + x\cos(x)) dx = \int \sin(x) dx + \int x\cos(x) dx$$

2. Integrate $\sin(x)$:

$$\int \sin(x) dx = -\cos(x) \quad \checkmark$$

3. Integrate $x\cos(x)$ using integration by parts: For $\int x\cos(x) dx$, we use integration by parts where:

- $u = x \implies du = dx$
- $dv = \cos(x) dx \implies v = \sin(x)$



Applying the integration by parts formula $\int u dv = uv - \int v du$:

$$\int x\cos(x) dx = x\sin(x) - \int \sin(x) dx \quad \checkmark$$

Now, integrate $\sin(x)$:

$$\int \sin(x) dx = -\cos(x) \quad \checkmark$$

AI: Microsoft Copilot

So, we have:

$$\int x\cos(x) dx = x\sin(x) + \cos(x) \quad \checkmark$$

4. Combine the results:

$$\int (\sin(x) + x\cos(x)) dx = -\cos(x) + x\sin(x) + \cos(x) + C \quad \checkmark$$

5. Simplify the expression:

$$\int (\sin(x) + x\cos(x)) dx = x\sin(x) + C \quad \checkmark$$

Therefore, the integral of $\sin(x) + x\cos(x) dx$ is:

$$\int (\sin(x) + x\cos(x)) dx = x\sin(x) + C$$

✓

where C is the constant of integration.

Isn't it satisfying to see how these integrals come together? If you have any more questions or need further clarification, feel free to ask!

Enzo Vozzo

After working as a Technical Officer at Telstra, Enzo graduated from Monash University in 2005 with a Bachelor of Technology (Computer Studies) and taught Electronics and Communications Engineering at Chisolm TAFE.

In 2013 he graduated from RMIT University with a Graduate Diploma of Education teaching Secondary School Mathematics and Science.

Since 2016 he has been teaching Mathematics at Mentone Grammar.



Email: exv@mentonegrammar.net



Instagram iphiepi: <https://www.instagram.com/iphiepi/> →

$$\begin{array}{ll} i = \sqrt{-1} & \phi = \frac{1 + \sqrt{5}}{2} \\ e = \sum_{n=0}^{\infty} \frac{1}{n!} & \pi = 4 \int_0^1 \sqrt{1-x^2} dx \end{array}$$



YouTube Channel: Maths Whenever:

https://www.youtube.com/channel/UCFLdfe_y2OQ1MZvGjha9taQ/videos

